

Application of TDA: 3D Segmentation

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Overview

These days, the amount of sensory data that has been collected has increased, and it is becoming increasingly hard to process this data for machine learning applications.

Through this project, I will present a Machine Learning application of Topological Data Analysis (TDA), a rapidly evolving field of data science which makes use of topology to improve this kind of data analysis.

Objective

Present a visual application of TDA in geometry processing, namely 3D shape segmentation.

Background

3D Segmentation

3D shapes are usually stored as a set of points, edges and triangles in a computer. The goal of segmentation is to provide labels for each point of each shape. For instance, if you are given a bunch of 3D shapes representing humans, the goal of segmentation is to successfully assign for each point the body part to which it belongs (“torso”, “arm”, “leg”...).

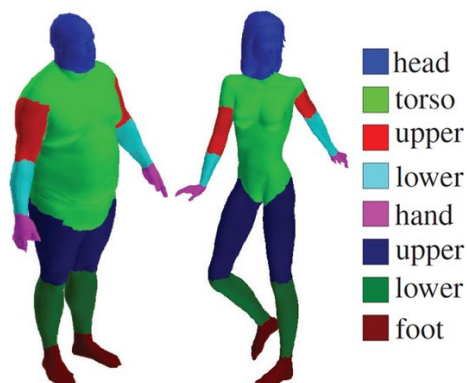


Fig 1: Visual Demonstration of 3D Segmentation

Topological Data Analysis

1. TDA is a mathematically grounded theory which aims at characterizing data using its topology, which is done by computing features of topological nature. The most common one is the persistence diagram, which takes the form of a set of points in the plane above the diagonal.

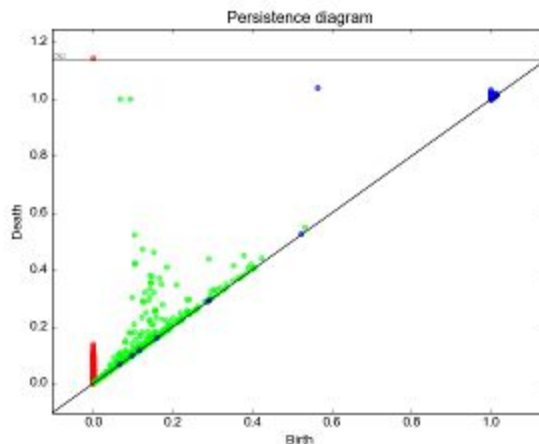


Fig 2: Example of a Persistence Diagram

2. Each of such point represents a topological feature (such as a connected component, a hole or a cavity) of the data. Moreover, the distance of the point to the diagonal act as an indicator of the importance of the corresponding feature, the usual interpretation being that points close to the diagonal are likely due to noise.
3. The computation of such diagrams requires a filtration, that is, a sequence of growing spaces: each space in the sequence is included in the next one. For instance, given a point cloud, a possible filtration would be to compute unions of balls centered on the points with a sequence of increasing radii.

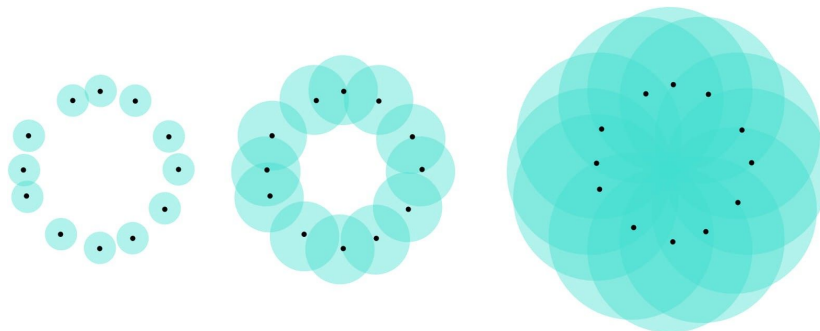


Fig 3: Visualization of the Filtration Process

Advantages of Topological Data Analysis

The idea is, for each space in the sequence, to record whether a topological feature is either created or destroyed in that space.

Three advantages of this process are:

- Coordinate Invariance
- Deformation Invariance
- Compressed representation

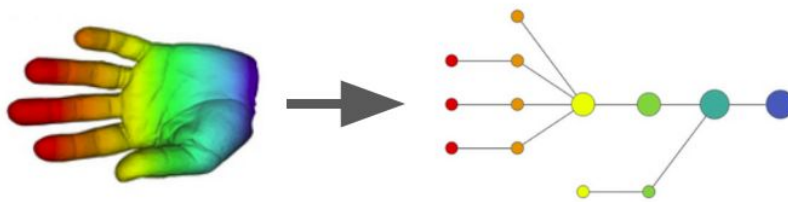


Fig 4: Visual Representation of the Process

Problem Associated With 3D Segmentation

- The difficulty of this problem lies in the fact that you are only given point coordinates, which are poor features.
- Indeed, it is hopeless to characterize a point with its coordinates since they depend on the embedding, or pose, of the 3D shape.
- Think for instance of two human shapes, one of them having its right hand raised and not the other; the humans are identical, only their poses differ. Then, the right hand points of the two shapes will differ a lot, even though they share the same label.

Solution: TDA to the Rescue!

Thanks to their topological nature, persistence diagrams are intrinsic, meaning that they do not depend on the embedding, or pose, of the 3D shapes. Hence, they are good candidates for point features. To do this we thus need to define an intrinsic filtration.

This can be achieved with geodesic distances.

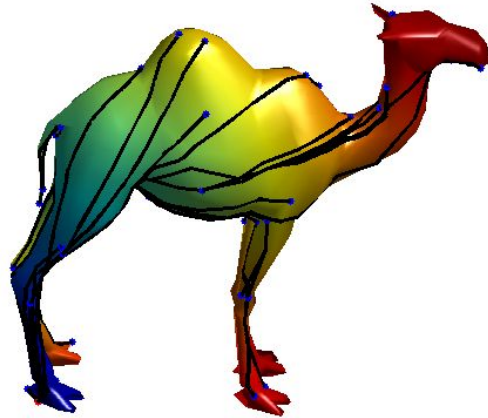


Fig 5: Visualization of Geodesic Distances

Application of TDA

Geodesic distances can then be used to define geodesic balls. Again, by making r increase from 0 to infinity, we make the geodesic ball grow from the singleton $\{x\}$ to the whole shape itself, which gives us an intrinsic filtration.

For example, take a look at the filtration displayed below on a 3D hand shape.

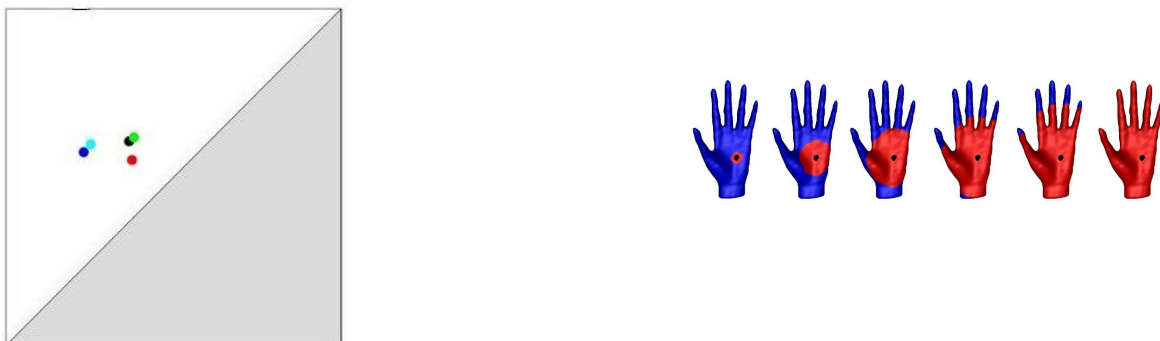


Fig 6: Mapping Persistence Diagram to Visualization of Filtration