

Design and Implementation of *Pax Mundi II*

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Abstract

On January 18, 2007 a ten foot tall bronze sculpture *Pax Mundi II* was installed in the courtyard of the H&R Block headquarters in Kansas City. This paper describes the computer-aided re-design process that started from the original *Pax Mundi* wood sculpture, as well as the fabrication and installation of the final sculpture.

1. Background

The three authors have enjoyed artistic collaboration since the mid 1990s. Collins has been carving organic looking abstract geometrical forms for more than two decades [1]. Reinmuth has been casting some of Collins' work in bronze and applying the finishing touches comprising some exquisite patinas. Séquin, inspired by some of these beautiful shapes (Fig.1a), contacted Collins in 1995 and they started to discuss the mathematical rules and symmetries inherent in Collins' work. They also discussed how these paradigms might get extended to produce novel and/or more complicated forms. A key challenge was to evaluate which of the many conceptual geometrical ideas would also have some aesthetic merits. The weekly lengthy phone conversations between Collins and Séquin generated far more ideas than could possibly be constructed even as models. Thus Séquin offered to create a generative program for one of the family of shapes and thereby introduce the computer as a visualization tool to judge the aesthetic merits of the various conceived constructions (Fig.1b). The result was *Sculpture Generator I*, a special purpose computer program for the design and visualization of Scherk-Collins hole-and-saddle chains warped into toroidal configurations [6][7]. Some of the promising looking designs were then realized as maquettes on rapid prototyping machines such as the Fused Deposition Modeling (FDM) machine from Stratasys [11]. A couple of designs, such as the *Hyperbolic Hexagon II*, and the *Heptoroid* or *Hyperbolic Heptagon II* (Fig.1c) were realized as larger wood sculptures, about 30 inches in diameter. Once Collins had decided what the final size should be, Séquin scaled the virtual model to that desired size and sliced its geometry at 7/8 inch intervals, corresponding to the board thickness from which Collins wanted to assemble the final wood sculpture. A stack of 3×3 foot size blueprints were then generated, which defined the detailed geometry at the proper slicing intervals. Collins cut out these rather convoluted shapes and assembled the

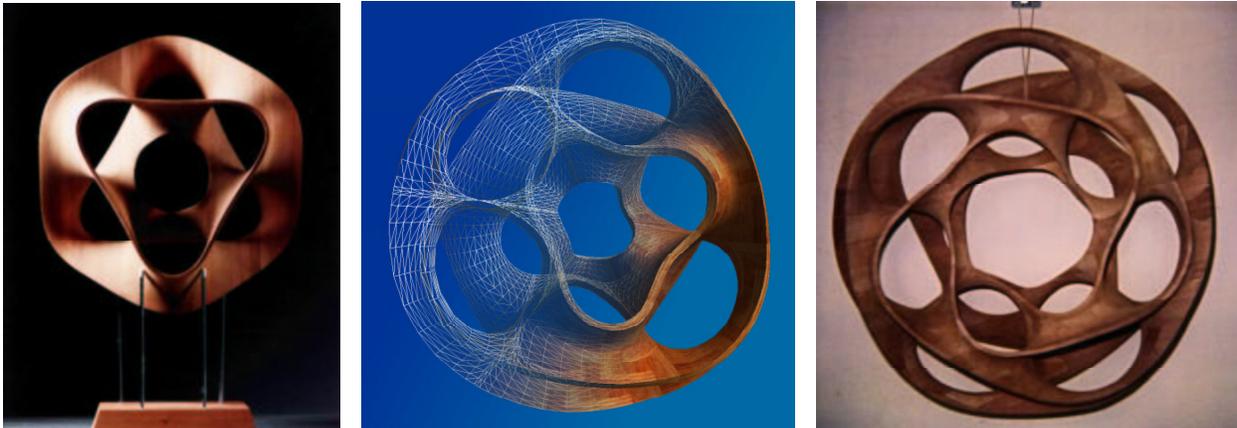


Figure 1: (a) *Hyperbolic Hexagon*, (b) *Hyperbolic Hexagon II*, (c) *Heptoroid* or *Hyperbolic Heptagon II*.

cut-out boards, properly registered to one another. In a 2-month long carving process he removed the stair-casing produced by the boundaries of the cut-out boards and smoothed the surface to perfection. The result clearly demonstrated that this collaboration between an artist and a computer scientist could be successful and produce sculptures that neither of them alone could have created (Fig.1c).

2. Collins' Pax Mundi

In 1997 Collins also carved *Pax Mundi* (*World Peace*), using a completely different geometrical paradigm [2]. This was an undulating ribbon skirting the surface of a two-foot virtual sphere (Fig.2a,b). Séquin was so taken by this sculpture that he decided to create a computer program that could emulate this shape and perhaps make other sculptures similar to it. The scope of *Sculpture Generator I* was much too narrowly focused; thus a completely new program had to be devised to capture the shape of *Pax Mundi* for this purpose (Fig.2c). At this juncture, the first and conceptually most challenging step is to understand the sculpture to be captured in terms of a broad enough paradigm, so that it can be easily modeled with convenient parameters, and so that the program lends itself to the generation of a wide variety of other related sculptures. From the onset it was clear that *Pax Mundi* should be modeled as a generalized sweep along a guide curve embedded in the surface of a sphere – following Collins' original concept. It was less clear how the sweep curve itself should be parameterized.



Figure 2: *Pax Mundi* by Brent Collins (1997): (a) wood master, (b) bronze cast by Steve Reinmuth; (c) emulation with Séquin's *Gabo Sweep Generator* programmed in SLIDE [10].

The breakthrough came when that sweep curve was viewed as a generalization of a baseball seam. The undulations are reminiscent also of the curved edges found in sculptures by Naum Gabo [4]. Thus Séquin formulated the concept of an *n-lobe Gabo curve* as a meandering path that oscillates around the equator of a sphere, with *n* lobes on either side of it. In its most symmetrical form, it has $2n$ regularly spaced inflection points along the equator and overall exhibits the symmetries of an antiprism, with symmetry group D_{nd} . From this perspective, tennis ball or baseball seams are just symmetrical 2-lobe Gabo curves.

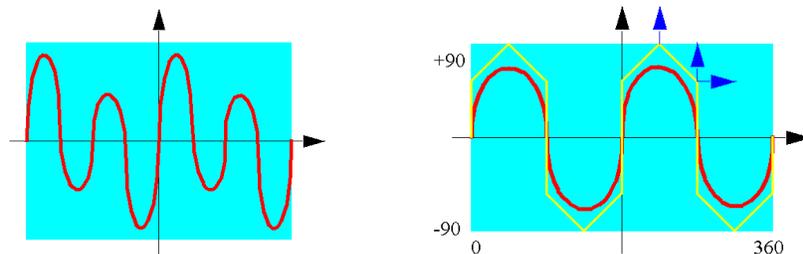


Figure 3: (a) The curve used to emulate *Pax Mundi*; (b) lobe shape control in the *Gabo curve generator*.

To create more artistically interesting sculptures, it appeared advantageous to reduce this symmetry and to permit asymmetrically bulging lobes, or to assign different heights to different lobes. In this light, *Pax Mundi* can now be understood as an amplitude-modulated 4-lobe Gabo curve (Fig.3a); every second lobe on each hemisphere is of lower amplitude. This leads to the characteristic pattern at the poles of *Pax Mundi*: Two opposing large lobes almost touch, while the two shorter, secondary lobes stay back from this junction.

Because of these considerations, the generator program for this type of ribbon sculpture was broken down into three modules, described in the next three paragraphs.

The Sweep Curve Generator. Originally this was a special purpose program module that allowed the user to specify the number of lobes, their heights, and their widths. A simple B-spline mapped onto the sphere was used. Three numeric parameters per lobe provided sufficient design freedom (small arrows in Fig.3b). Several different lobes can be tied together to the same set of parameters to maintain some regularity or symmetry and to make the design process less laborious. At a later development stage, an additional parameter was added that allowed bending the tips of the lobes ever so slightly outwards or inwards towards the center of the sphere. This adjustment was deemed important to allow for the precise visual alignment of the lobes congregating around one of the poles.

The Cross Section Fine-Tuner. This is a plug-in module that generates a cross sectional profile of a particular type. In the case of *Pax Mundi* the profile is a crescent with rounded points. A few sliders allow to adjust the size of the crescent, the arc-angle of the dominant inner and outer curves as well as the radius at the tips. All these parameters typically need to be re-adjusted as the sculpture is scaled to different sizes or when the fabrication process is changed.

The Sweep / Twist Controller. This module controls the orientation of the cross section along the sweep path. By default, the cross-sectional profile lies in a fixed orientation in the normal plane of the Frenet frame of the sweep curve (Fig.4a). This is the case for the *Pax Mundi* sculpture. The concave part of the crescent profile always opens opposite to the local normal vector of the curve. Thus the local curvature of the sweep curve together with the concave part of the profile will produce a continuous elongated saddle of negative Gaussian curvature. This yields an aesthetically pleasing geometry and also provides extra stability for this extended ribbon sculpture.

Alternatively, we can select the option to minimize torsion. Now the orientation of the cross section is no longer tied to the Frenet frame, but instead is adjusted in a progressive way so as to minimize the torsion of the ribbon. A small amount of overall twist is introduced to achieve seamless end-to-end closure of the ribbon. For our particular sweep curve, a nice and consistent shape emerges, in which the ribbon is mostly tangential to the sphere (Fig.4b). If we change the azimuth value by 90 degrees, the ribbon takes on a radial orientation (Fig.4c) and it can now be widened considerably without creating self-intersections. The result looks very much like an original Gabo surface [4].

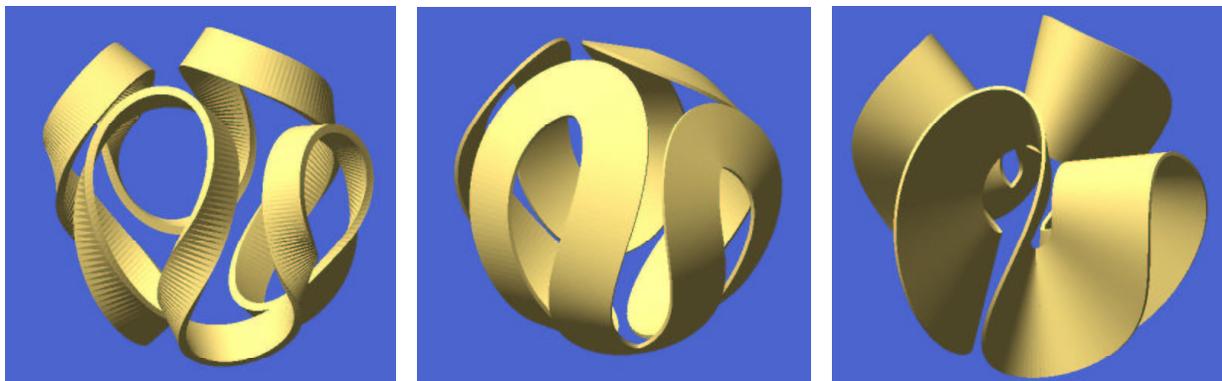


Figure 4: Sweep control options: (a) cross section follows Frenet frame; (b) torsion minimization with an azimuth value that keeps the ribbon mostly tangential to the base sphere, and (c) normal to it.

This new modeling system has much more versatility than is needed for the emulation of *Pax Mundi*. First, this sweep generator permits one to control the rotation of the profile as it sweeps along the guide path – either in the form of a continuous twist parameter, or, more locally, by specifying the azimuthal orientation for selected cross sections at some of the control points of the curve; in the latter case, these azimuthal angles get interpolated in exactly the same manner as the x-, y-, and z-coordinates of the control points themselves. The sweep controller also allows the specification of affine deformations of the basic cross sectional profile for each of the curve control points – which again get interpolated by the formula used for the coordinates and the azimuth values. The sweep generator can even morph one profile into another one – provided they both have been described with the same number of data points.

At a later time the capabilities of this system were further extended. First a curve editor was created that allowed one to draw more general undulating pathways onto the base sphere [9]. This resulted in a set of *Viae Globi (Pathways on a Sphere)* sculptures [8]. One of those maquettes (Fig.5a) inadvertently exhibited some striking similarities with Collins’ sculpture *Egg* [3], even though it was generated with a completely different starting paradigm (three sets of parallel traces on opposite sides of a cube).

The next step of generalization lets the sweep path deviate from the surface of the sphere. Collins has already carved a sculpture in this category, called *Music of the Spheres* [3]. Séquin’s modeling environment could easily be extended to capture this particular sculpture; he just had to use a more general 3D curve editor to produce the original sweep curve. The path of *Music of the Spheres* stays mostly on the sphere; it just makes three excursions that dive into the interior of the sphere to form three additional loops (Fig.5b). With the same generator we can also produce more complicated 3D paths – for instance some that are truly knotted, such as the *Chinese Button Knot* (Fig.5c).



Figure 5: Different sweep paths: (a) *Altamont* [8]; (b) *Music of the Spheres* [3]; (c) *Chinese Button Knot*.

3. Scaling Up

The dream of most artists is to implement their work on a grand scale in some public location, where it can be seen by thousands of people. Realizing such sculptures presents a whole new set of problems. In 2006 we had the opportunity to face these new challenges, when Collins received a commission to scale up *Pax Mundi* to a bronze sculpture of about 6 feet in diameter as a signature piece for the H&R Block headquarters building in Kansas City.

It was immediately clear that such a large sculpture could not be cast as a single piece. A small extension to Séquin’s generator program made it possible to decompose the sculpture into four copies of a single master shape in the form of giant 3-dimensional S-curve (Fig.6a,b). However, the degree of the sweep curve spline had to be increased and its control points had to be regrouped and relocated so as to obtain clean boundary conditions at the joints between subsequent segments. Since we anticipated that

welding together the individual ribbon sections in a perfectly straight continuous manner would be difficult, alignment tabs were added at the joints. These allow Reinmuth to clamp together the individual pieces in perfect alignment during the assembly process. One of these junctions had an extra large tab (Fig.7b) which is kept as an integral part of the assembly and becomes the support of the sculpture on its pedestal.



Figure 6: (a) *Pax Mundi* emulation with four identical S-shaped segments; (b) prototype of a single master segment; (c) S-shaped segment split into two U-shaped parts.

Reducing the number of master patterns and molds that need to be generated is an important economic consideration, since it is a relatively expensive and time consuming part of the process. The machined master patterns are used to make negative molds of plastic and silicone, which in turn are then used to cast multiple positive copies in wax. These wax replicas of the master shape are then sacrificed in an ordinary investment casting process, which creates the four bronze copies needed to assemble the whole sculpture.



Figure 7: (a) The milling machine used to produce the master patterns; (b) the smaller U-shaped master segment obtained by CNC milling (Photos supplied by Joe Valasek at heartwoodcarving.com).

At this point we faced the problem of how to create the master pattern. No rapid-prototyping machine based on layered manufacturing was available at affordable prices to make the S-shaped master pattern in one piece. Eventually we found an opportunity to create such master patterns at economical rates on a computerized numerically controlled (CNC) milling machine (Fig.7a), using hard, dense foam plastic. However, this machine had a limited clearance that could not handle the bulky S-shaped master in one

piece. By cutting that shape at the inflection point, we obtained two much flatter U-shaped sub-master patterns (Fig.6c) that could just fit into the machine. Extra alignment tabs were added (Fig.7b) to accommodate the extra junctions and to help in the final assembly.



Figure 8: (a) Two sub-segments of the partitioned U-shaped master patterns and their molds; (b) two wax copies of such sub-segments with sprues and risers.

4. Bronze Casting

Reinmuth Bronze Studio [5] was selected to cast the pieces and assemble the final sculpture. It turned out that the milled master patterns, the smaller of which is shown in Figure 7b, were still too large for convenient handling in the bronze foundry. Thus Steve Reinmuth cut the smaller U-shape into two halves, and the larger master pattern into three pieces. Before cutting the master patterns apart, Reinmuth made a special jig that would allow him later to recreate those U-shapes perfectly from the individual pieces, even though there were no alignment tabs at the extra junctions introduced by him.



Figure 9: (a) Ceramic shell around wax copies; (b) casting process; (c) cooling bronze in the risers.

The master patterns were used to create silicone molds (Fig.8a) in which Reinmuth could then cast multiple copies in wax. The wax copies were outfitted with the necessary sprues and runners to distribute the liquid bronze quickly to the whole mold cavity and were also provided with risers (funnels) into which the melt could be poured easily (Figs.8b, 9b). The enhanced wax segments were then repeatedly dipped into a ceramic slurry, until a solid shell had been formed around the whole part (Fig.9a). These ceramic shells are put into an oven, preheated to 1800°F, where they sinter into hard forms and where the

wax drains out. The fired shells are then filled with liquid bronze of about 2100°F. After the metal has cooled down and hardened, the ceramic shell is cracked into small fragments and removed (Fig.10a).

5. Assembly and Shape Adjustment

The individual pieces, of which there were twenty for *Pax Mundi II*, are cleaned and freed of the extraneous sprues and risers. Then the assembly process begins. Some of the smaller pieces are assembled into the U-shapes of the original master patterns. Then the overall assembly process continues in a bottom-up manner, starting from the point of support (Fig.10b), and progressing in a symmetrical manner to guarantee perfect symmetry and balance of the overall sculpture.



Figure 10: (a) A raw cast piece after ceramic shell has been removed; (b) two of the cast pieces welded together.

Since the *Pax Mundi II* sculpture is essentially one long loop that is supported at just one point, the top part of this heavy ribbon sagged by almost one foot under its own weight. To compensate for that, Reinmuth had to pre-distort the shape during the assembly process. He hung the sculpture upside-down and let it elongate under its own weight (Fig.11a). He enhanced that elongation by cutting several slits into the ribbon (Fig.11b) so that some tight bends could open up. Then he refilled the enlarged slits with bronze welds. This locked in an elongated state of the sphere, which then settled to an almost perfectly spherical shape when the sculpture was put back into its normal position.



Figure 11: (a) Adjusting the overall shape (b) by cutting the ribbon partly and infilling more material.

After the overall shape had been adjusted to perfection, the many welded junctions and adjustment slits had to be ground down to seamlessly blend into the flow of the ribbon. Finally a tan-colored patina was

applied with a judicious combination of heat and chemicals (Fig.12a). This color is protected from further oxidation with a thin coating of wax. As long as that coating is refreshed every few years by rubbing over the whole surface of the sculpture with a block of wax on a hot day, it should maintain its current color (Fig.12b) for a very long time.

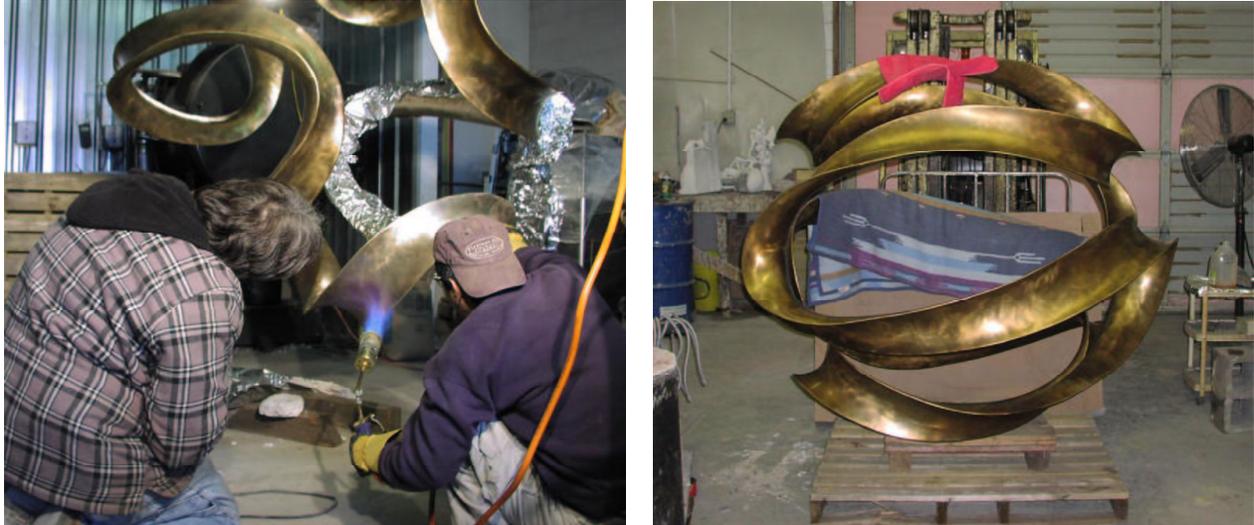


Figure 12: (a) Applying patina to the surface. (b) *Pax Mundi II* ready for shipping

6. Installation

Finally, in mid January the sculpture was finished and could be shipped to Kansas City. In the morning of January 18, 2007 we picked it up from the loading dock at H&R Block, carefully transported it around the building (Fig.13a), and maneuvered it through the main entrance (Fig.13b) by just two inches! We wheeled the sculpture through the main lobby to the courtyard in the back of the building, where a stainless steel pedestal and a scaffolding with a small crane were waiting (Fig.13c). After a few anxious moments, due to some difficulties with the mounting bolts, *Pax Mundi II* was in its designated location (Fig.14a,b).



Figure 13: (a) Transport around the H&R Block building, (b) through its lobby, (c) to the courtyard.



Figure 14: (a) Installation on pedestal; (b) *Pax Mundi II* in its final location.

7. Conclusions

Pax Mundi II is the result of a truly collaborative effort among the three authors – none could have created this piece of art by himself. The one character trait that all three authors have in common is that we are perfectionists. We all are willing to go repeatedly through some labor-intensive and possibly tedious process to guarantee that the result is as good as it possibly can be.

Ten years ago Brent Collins first calculated and drew painstakingly the different slices of the hollow wooden sphere from which the original *Pax Mundi* sculpture was carved, and then carefully designed and shaped the sinuous sweep path on that globe.

In the summer of 2006, Carlo Séquin spent hours adjusting the many parameters in his Gabo-curve generator program to match the spacings and curvatures of the original sculpture based on just four photographs that he had available. He also fine tuned the exact cross section of the ribbon several times while consulting closely with Collins about aesthetic issues and with Reinmuth about stability, weight and volume concerns.

The final shape control was then entirely in Steve Reinmuth's hands. First he fine-tuned and finished the CNC-milled master patterns, defining the geometry in those places that the 3-axis milling machine could not reach. Subsequently he had to address the unexpected sagging of the ribbon under its own weight. Several tries were necessary to establish the best places to introduce the subtle curvature changes that overall would compensate for the effects of gravity. Subsequently he ground and polished to perfection the twenty junctions where two individually cast ribbon sections had to be joined, as well as the places where he made partial cuts and in-fills to correct the overall shape. Finally he added the beautiful patina that gives this sculpture its refined look.

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