Hilbert Cube 512

Carlo H. Séquin*
CS Division, U.C. Berkeley

1 The Design: From 2D to 3D

“Hilbert Cube” emerged from the challenge of taking the famous 2-dimensional Hilbert curve (Fig.1) and exploring what can be done with this pattern in 3 dimensions. The 3D shape is generated by a recursive procedure that starts with a simple closed path along eight edges of a cube visiting all its eight vertices. Each corner in this structure is then replaced with a copy of this path, scaled down by a factor of 2, and suitably connected to maintain the overall cyclic nature of the path. After three recursion steps, a structure emerges with a total of 512 L-shaped turns. The result resembles a cubist rendering of a brain, split into two distinct lobes that are only loosely connected to one another. These halves are again divided into two loosely connected halves, and so on.

There were many challenges in realizing the initial vague concept. One of the overriding concerns was to obtain maximum symmetry in this construction, and, unlike the original Hilbert curve, I wanted the overall loop to close back onto itself. Also unlike the 2D curve, I did not want to allow any consecutive collinear segments. I even took great care to ensure that no more than two consecutive “L”s ever lie in the same plane (Fig.2). More details of the constructive logic will be presented in the artist talk. Many combinations of splitting, twisting, and assembly of the individual recursive modules had to be tried out to meet all mathematical and aesthetic requirements. This would not have been possible without the help of computer-aided tools.

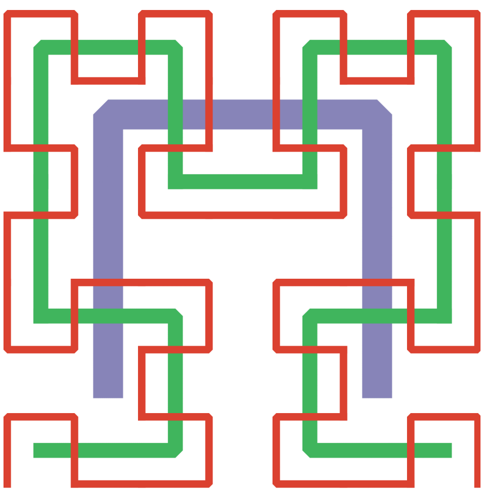

Figure 1. Three generations of the recursive Hilbert curve.

2 Implementation: Novel Metal Sintering Process

Realization as a physical sculpture also presented formidable challenges. The 3D Hilbert curve at the 3rd level of recursion is essentially a rod 3/16th of an inch thick and 320 inches long, wound up in a 4-inch cube! A material is needed that can carry its own weight and hold its shape over the whole length of the rod with no intermediate supports. A realization in metal is clearly desirable. But how would one cast such a complicated shape? Machining is not an option either, since the milling tool could not gain access to the inner parts of this sculpture.

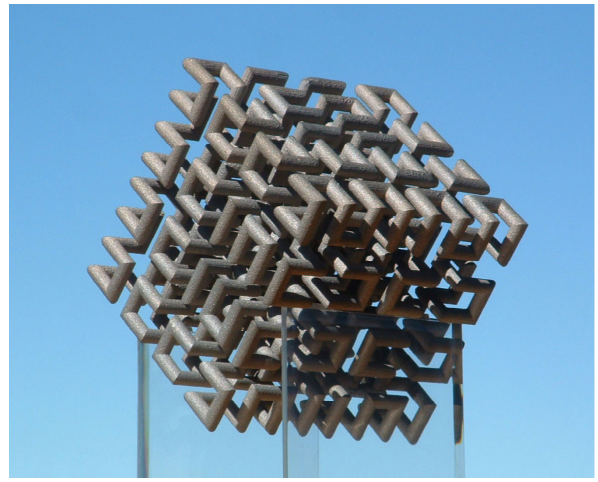

Figure 2. Hilbert Cube in stainless steel and bronze.

Fortunately, a suitable rapid prototyping process recently became available from ProMetal, a division of ‘The Ex One Company’ headquartered in Irwin, PA. In this process a ‘green’ part is first formed, composed of stainless steel powder and a selectively applied binder. This part is built bottom-up, layer-by-layer under direct computer control; only a boundary representation of the desired shape is required as input. This green part is then sintered, and the binder is drained out and replaced by liquid bronze. This makes it possible to fabricate very complex parts with no need for molds or for machining.

However, Hilbert Cube presented a tough challenge even for this advanced process. The ‘green’ part does not have the strength of the final metal part, and without additional supports it would sag and crumble under its own weight. 36 additional supports were required to hold the part rigid through the sintering process. Since these supports had to be cut away in the final metal structure, they had to be located near the periphery of the cube, where they were more easily accessible. But some supports were also needed near the center of the structure. Since these were almost impossible to remove, they had to be part of the desired final Hilbert pattern. This required one more final re-design of the generative procedure to move the connections between the two halves of the brain into the center of the cube.

3 Finding a Generative Paradigm

Hilbert Cube is only one instance of an intriguing structure that was found by asking: “How could this be done in 3 dimensions?” Other examples started with the 2D Yin-Yang symbol or with a simple logarithmic spiral. Another key element of my work is to look for the inherent constructive logic of an elegant geometric sculpture and then devising an appropriate generative paradigm. If such a procedure can be found and captured in a computer program, then, with only a moderate effort, many more artistic instances belonging to the same conceptual sculpture family can be generated. This approach has been successfully applied to a couple of masterpieces by Brent Collins and has led to a stimulating and productive collaboration. Examples will be shown in the artist talk.

* e-mail: sequin@cs.berkeley.edu