1. Show that Markov's inequality is tight: namely,
   (a) Give an example of a non-negative r.v. X and a value $k > 1$ such that $\Pr[X \geq kE[X]] = \frac{1}{k}$.
   (b) Give an example of a r.v. X (with $E[X] > 0$) and a value $k > 1$ such that $\Pr[X \geq kE[X]] > 1/k$.

2. We revisit a problem from last week: Suppose we flip a fair coin $n \geq 3$ times to obtain n random bits. Consider all $m = \binom{n}{2}$ pairs of these bits in some order. Let Y_i be the exclusive-or of the i'th pair of bits, and let $Y = \sum_{i=1}^{m} Y_i$ be the number of Y_i's that equal 1.
   (a) Show (again) that $E[Y] = \frac{m}{2}$ and $\mathrm{Var}(Y) = \frac{m}{4}$.
   (b) Use Chebyshev's inequality to derive an upper bound on $\Pr[|Y - \frac{m}{2}| \geq n]$.
   (c) Let $p = \Pr[Y \leq \frac{3}{4}m]$.
      i. Using Markov's inequality, show that $p \geq \frac{1}{3}$.
      ii. Using Chebyshev's inequality, show that $p = 1 - O(\frac{1}{m})$.
   (d) Consider the case $Y = 0$ or $Y = m$.
      i. Using Chebyshev's inequality, show that $\Pr[Y = 0 \vee Y = m] \leq 1/m$.
      ii. Find all possible sequences of n coin tosses such that $Y = 0$ or $Y = m$.
      iii. What is the exact value of $\Pr[Y = 0 \vee Y = m]$?