

1. Consider the event that every bin receives exactly 2 balls when  $2n$  balls are thrown randomly into  $n$  bins.<sup>1</sup>
  - (a) Compute the probability under the Poisson approximation.
  - (b) Determine the exact probability of this event.
  - (c) The value upon dividing the expression in (a) by that in (b) equals the probability that a Poisson r.v. with parameter  $\lambda$  takes on some value  $k$ . What are the values of  $\lambda$  and  $k$ ?
  
2. Suppose that we vary the balls-and-bins process as follows. For convenience, let the bins be numbered from 0 to  $n - 1$ . There are  $\log n$  players. Each player randomly chooses a starting location  $\ell$  uniformly from  $[0, n - 1]$  and then places one ball in each of the bins numbered  $\ell \bmod n, \ell + 1 \bmod n, \dots, \ell + n/\log n - 1 \bmod n$ .
  - (a) Show that the probability that bin 0 receives at least  $k$  balls is at most  $(e/k)^k$ .
  - (b) Show that the maximum load is  $O(\log \log n / \log \log \log n)$  with probability  $1 - o(1)$ .<sup>2</sup>  
[HINT: what happens when  $k = 3 \ln \log n / \ln \ln \log n$ ?]

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<sup>1</sup>The answers are (a)  $(2e^{-2})^n$  (b)  $\frac{(2n)!}{2^n n! 2^n}$  (c)  $\lambda = k = 2n$  (compare with MU Theorem 5.6).

<sup>2</sup>The key observation is that we do not need to take a union bound over all  $n$  balls; it suffices to take a union bound over the  $\log n$  bins that are chosen by the players. In particular, if the maximum load is more than  $k = 3 \ln \log n / \ln \ln \log n$ , then one of the  $\log n$  bins chosen by the players has load more than  $k$ .