

Homework 8

Out: 25 Oct. Due: 1 Nov.

Instructions: Start each problem on a new sheet. Write your name, SID number, section number and “CS70” on every sheet. Clip together (do not staple!) the sheets for different problems (but if you use more than one sheet for one problem, staple those). Put your solutions in the homework box on Soda level 2 by 5pm on Thursday. You are encouraged to form small groups (two to four people) to work through the homework, but you **must** write up all your solutions on your own.

1. Palindromic numbers

A number is a *palindrome* if it reads the same forwards as backwards. (So, e.g., 13531 and 9559 are palindromes, but 14631 is not.)

- How many n -digit palindromes are there? [NOTES: You may consider separately the cases that n is odd and even. An n -digit number cannot begin with a 0.]
- An n -digit number is chosen uniformly at random. What is the probability that it is a palindrome?

2. Poisoned pancakes

You have been hired as an actuary by IHOP corporate headquarters, and have been handed a report from Corporate Intelligence that indicates that a covert team of ninjas hired by Denny’s will sneak into some IHOP, and will have time to poison five of the pancakes being prepared (they can’t stay any longer to avoid being discovered by Pancake Security). Given that an IHOP kitchen has 50 pancakes being prepared, and there are ten patrons, each ordering five pancakes (which are chosen uniformly at random from the pancakes in the kitchen), calculate the probabilities that a particular patron:

- will not receive any poisoned pancakes;
- will receive exactly one poisoned pancake;
- will receive at least one poisoned pancake.

3. A quickie

A man lives in a house with a railroad track at the end of his garden. The rail line connects Pleasantville with Happytown, and carries no other traffic. Trains run all day at exactly one hour intervals from Pleasantville to Happytown and vice versa. Just for fun, the man goes down to the bottom of his garden at a random time every day and waits until the first train passes. After a month, he notices that the first train was from Pleasantville on 20 of the days, and from Happytown on the other 10 days. How can this be? Explain your answer carefully.

4. Monty Hall revisited

In this version of the Monty Hall problem, after the contestant has chosen a door, Monty asks another contestant to open one of the other two doors. That contestant, who also has no idea where the prize is, opens one of those two remaining doors at random. If he happens to open the door with the car, then he receives the car and you receive nothing. If he opens a door with a goat, then Monty asks you if you wish to switch doors or stick with your original choice. What is your best strategy now? Explain your answer carefully, with reference to the appropriate sample space. (I.e., give an analysis similar to the one in the notes for the original problem.)

[continued overleaf]

5. Colorful coins

We are given three coins. The first coin is a fair coin painted blue on the heads side and white on the tails side. The other two coins are biased so that the probability of heads is p . They are painted blue on the tails side and red on the heads side. Two of the three coins are selected at random and flipped.

- (a) Describe the outcomes in the sample space, and give their probabilities. [NOTE: You may want to draw a tree to illustrate the sample space.]
- (b) It was determined experimentally (by repeating the above experiment many times) that the probability that the two sides that land face up are the same color is $\frac{29}{96}$. What does this tell you about the possible values of p ?

6. A good bet?

Your friend proposes the following game. She will roll six fair dice. If the number of *different* numbers that show up is exactly four, then you win \$1. Otherwise, she wins \$1. Would you bet on this game? Justify your answer with a calculation. [HINT: Be very careful in counting the number of ways that exactly four distinct numbers can show up! You should find that the game is rather finely balanced!]