

CS 70 FALL 2007 — DISCUSSION #6

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1. DEFINITIONS

Exercise 1. Find a de Bruijn sequence for $n = 4$.

2. APPLICATIONS OF GRAPHS

Exercise 2. Formulate each of the following problems as graph problems; in each case, say whether the graph is directed or undirected, and say what property or structure of the graph one needs to discover.

- (1) Finding one's way out of a maze
- (2) Figuring out an order in which to assemble a complex piece of machinery given that certain sub-parts have to be assembled before others are begun
- (3) Finding the shortest (in mileage) route from SF to LA
- (4) Pairing up a bunch of n boys and n girls for a prom dance so that only people who know each other are paired up.
- (5) Detecting an *arbitrage opportunity* in the stock market (i.e., a sequence of buy/sell trades that, if all executed, yield a profit)

3. EULERIAN PATHS AND CIRCUITS

Exercise 3. For which n does the n -dimensional hypercube have an Eulerian circuit? How about an Eulerian path?

Exercise 4. A complete graph on n vertices, denoted K_n , is the graph containing all possible edges among n vertices. A complete bipartite graph on $m + n$ vertices, $K_{m,n}$ is the graph containing all possible edges such that one endpoint is from the set of m vertices and the other endpoint is from the set of n vertices. For which n does the complete graph K_n have an Eulerian circuit? For which m and n does the complete bipartite graph $K_{m,n}$ have an Eulerian circuit?

4. TREES

Exercise 5. A tree is a connected graph with no cycles. Show that any tree on n vertices has exactly $n - 1$ edges.

Exercise 6. Let G be any connected graph. A spanning tree is a subgraph of G without cycles that connects all vertices of G . Design an algorithm to find a spanning tree of G .