

1. Show that if you can factor  $N$  then you can efficiently and classically find the period of a periodic function mod  $N$ .
2. Show that phase estimation is complete for BQP.
3. Let  $v$  be the maximally entangled state on a  $d$  dimensional space  $H$ , i.e.  $v = \frac{1}{\sqrt{d}} \sum_{i=1}^d |i\rangle |i\rangle$ . Given two unitaries  $U, V$  acting on  $H$ , conclusively establish the relationship between these two quantities:  $\langle (U \otimes V)v, v \rangle$  and  $\text{tr}(UV)$ , using tensor networks. (I.e. your thinking and proof should feature pictures!).
4. a. Recall that given a vector  $v \in H \otimes H$ , the Schmidt decomposition of  $v$  is of the following form:
  - $v = \sum_i d_i a_i \otimes b_i$ ,  $a_i, b_i \in H$  and  $d_i$  non-negative numbers, such that
  - $\{a_i\}$  and  $\{b_i\}$  are each orthonormal sets.

Draw the tensor network picture of the Schmidt decomposition and write down (as pictures) the conditions for orthonormality.

- b. Suppose you have a decomposition of the form  $v = \sum_i f_i c_i \otimes d_i$ , such that  $\{c_i\}$  and  $\{d_i\}$  are orthogonal sets but not normal, and  $f_i$  are complex numbers. Can you easily realize this as a Schmidt decomposition. . . how do you do that?