1. **Teleportation** In the final step in quantum teleportation, player B performs one of four transformations depending upon the two classical bits received to obtain the quantum bit $\alpha|0\rangle + \beta|1\rangle$. One of the four cases is the identity transformation. The other three cases each correspond to transforming one of the following quantum bits to the desired bit $\alpha|0\rangle + \beta|1\rangle$. Give quantum circuits to effect each of these transformations.

   - $\beta|0\rangle + \alpha|1\rangle$.
   - $-\beta|0\rangle + \alpha|1\rangle$.
   - $\alpha|0\rangle - \beta|1\rangle$.

2. **Dirac Notation**: $|01\rangle$ is often alternatively written as $|0,1\rangle$, $|0\rangle|1\rangle$ or $|0\rangle \otimes |1\rangle$. The tensor product operator $\otimes$ is associative and distributive. Let $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. What is $|\psi\rangle^{\otimes 4} \equiv |\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle$? How is it related to

$$|\phi\rangle = \frac{1}{\sqrt{q}} \sum_{a=0}^{q-1} |a\rangle$$

   for $q = 2^4$?

3. **Shor State** Many quantum algorithms produce states of the form $\sum_a |a\rangle|f(a)\rangle$. An important step in Peter Shor's quantum factoring algorithm creates the state

$$|S(N,x)\rangle = \frac{1}{\sqrt{q}} \sum_{a=0}^{q-1} |a\rangle|x^a \bmod N\rangle.$$

   Let $x = 6$, $N = 91$, and $q = 512$. What state $|S'\rangle$ is obtained after measuring the first label (the one in the left ket) if a measurement result of $a = 34$ is obtained? *Extra Credit:* how does this result give the prime factors of N?

4. **BV rotations** Suppose that you have a single rotation gate that effects a rotation by angle $\theta = 2\pi \sum_i \frac{1}{2^{2^i}}$. Show that given any angle ϕ and an $\epsilon > 0$, how to use this single rotation gate to effect a rotation through an angle ϕ' such that $\phi - \epsilon < \phi' < \phi + \epsilon$. Bound the size of the resulting quantum circuit as a function of ϵ.

5. **Variants on teleportation** Start with two quantum bits in the state $\alpha|00\rangle + \beta|11\rangle$. Subject the first bit to a Hadamard gate. Now measure the first bit. What is the state of the second bit?