

## General Semi-separable Representation

Block  $4 \times 4$  example

$$A = \begin{pmatrix} D_1 & U_1 V_2^H & U_1 W_2 V_3^H & U_1 W_2 W_3 V_4^H \\ P_2 Q_1^H & D_2 & U_2 V_3^H & U_2 W_3 V_4^H \\ P_3 R_2 Q_1^H & P_3 Q_2^H & D_3 & U_3 V_4^H \\ P_4 R_3 R_2 Q_1^H & P_4 R_3 Q_2^H & P_4 Q_3^H & D_4 \end{pmatrix}.$$

In general

$$A_{ij} = U_i W_{i+1} W_{i+2} \cdots W_{j-1} V_j^H, \quad \text{if } j > i$$

$$A_{ij} = P_i R_{i-1} R_{i-2} \cdots R_{j+1} Q_j^H, \quad \text{if } j < i$$

## Low-rank blocks

Look at the sub-matrix  $H_2$

$$\begin{pmatrix} U_1 W_2 V_3^H & U_1 W_2 W_3 V_4^H \\ U_2 V_3^H & U_2 W_3 V_4^H \end{pmatrix} = \begin{pmatrix} U_1 W_2 \\ U_2 \end{pmatrix} \begin{pmatrix} V_3^H & W_3 V_4^H \end{pmatrix}$$

More generally every upper off-diagonal Hankel block in this representation has the form

$$\begin{pmatrix} U_1 W_2 W_3 \cdots W_i \\ U_2 W_3 \cdots W_i \\ \vdots \\ U_i \end{pmatrix} \begin{pmatrix} V_{i+1}^H & W_{i+1} V_{i+2}^H & \cdots & W_{i+1} W_{i+2} \cdots W_{n-1} V_n^H \end{pmatrix}$$

The low-rank of every off-diagonal Hankel block can be captured in this representation.

## Integral Equations on a curve

Discretizations of integral equations of the form

$$x(t) + \int_0^1 k(t, s)x(s)ds = b(t)$$

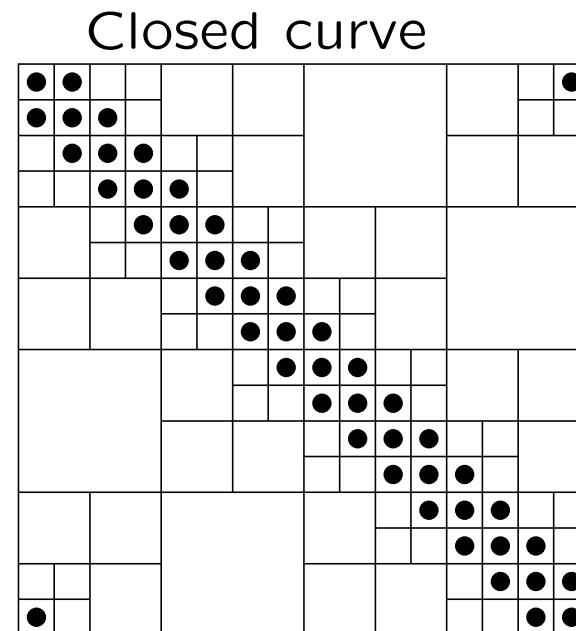
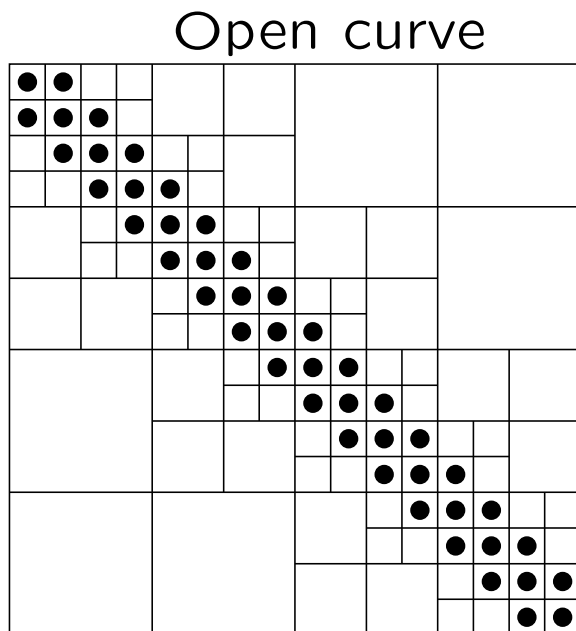
where  $k(t, s)$  is smooth away from the diagonal, leads to matrices with short representations.

Example kernels

$$\log \|x - y\| \qquad \frac{1}{\|x - y\|^r}$$

where  $x$  and  $y$  are points on a curve.

## Fine structure of off-diagonal ranks



For sequential elimination only Hankel-block ranks matter.

## Run-time in seconds

	size					
$m_i = k_i$	256	512	1024	2048	4096	8192
16	0.04	0.08	0.16	0.36	0.67	1.34
32	0.08	0.19	0.42	0.83	1.66	3.44
64	0.18	0.48	1.12	2.36	4.8	9.87
128	0.15	1.01	2.73	6.09	12.91	26.9
GEPP	0.15	0.72	4.57	30.46	222.57	3499.46

# Final Statements

- Representations are purely algebraic .
- Includes both dense and sparse matrices.
- Great potential for pre-conditioners.
- Interesting relationships with nested dissection and other sparse matrix techniques.
- Useful applications in a number of other important areas.