Math. 55 EXAM MODEL SOLUTIONS

This 25 minute exam, to be administered in your discussion section, can be answered with the aid of any texts, notes and calculating instruments. Answers may be worked out on scratch paper but must be entered on this sheet in the spaces provided. Each correct answer earns one point; each incorrect answer loses one point; space left blank loses nothing, so **DON'T JUST GUESS.** And check your work. Finally hand this sheet and *all* your used scratch paper to the Teaching Assistant, who will return the scored sheet some time later. Your score *will* affect your final grade. You must not discuss this exam's questions with anyone else until tomorrow.

1. $\sum_{j=5}^{7} a_j = a_5 + a_6 + a_7$. If $\sum_{j=k}^{n} a_j = \sum_{j=k}^{m} a_j + \sum_{j=m+1}^{n} a_j$ for *all* integers k, m and n, what must " $\sum_{j=7}^{5} a_j$ " mean? $\sum_{j=7}^{5} a_j$ = ________ $-a_6$ ____________ .

2. For every integer $k > 0$ let the product $\mu_k(x) := x(x-1)(x-2)(\dots)(x-k+2)(x-k+1)$, and set $\mu_0(x) := 1$. First find a compact expression for

$\mu_k(x+1) - \mu_k(x)$ = _ k _ μ_ $k-1$ _(_ x _) . (One point)

Then, for arbitrary integers m and $n \ge m$, find a compact expression with no "Σ" nor "…"

for $\sum_{j=m}^{n} \mu_k(j)$ = _ $(\mu_{k+1}(n+1) - \mu_{k+1}(m))/(k+1)$ _ .

3. The big–O notation defined on p. 80 of the text is used in the following propositions:

p(f) := " f(x) is $O(x^2)$ as x —› +∞ ." q(f) := " log(|f(x)|) is $O(\log(x))$ as x —› +∞ ."

r := " $\forall f$ p(f) → q(f) ." s := " $\forall f$ q(f) → p(f) ."

Is proposition r true? (YES or NO) _ YES _. Is proposition s true? (YES or NO) _ NO _.

Counterexample: $f(x) := x^3$.

4. Say for each of the following sets whether it is countable (YES) or not (NO) :

- All real numbers whose decimal representations contain no digit other than " 3 " : YES .
- All real numbers whose decimal representations contain no digits other than " 3 " or " 5 " : _ NO _.

Maximum possible score: 7 points.

Prof. W. Kahan Wed. 10 Feb. 1999