Math. 55 TEST Student's NAME in BLOCK CAPITALS: ___ Model Answers ___

This 20 minute test, to be administered in your discussion section, is to be answered without aid from texts, notes nor calculating instruments. Answers may be worked out on scratch paper but must be entered on this sheet in the spaces provided. Each correct answer earns one point; each incorrect answer loses one point; space left blank loses nothing, so **DON'T JUST GUESS.** Finally hand this sheet and *all* your used scratch paper to the Teaching Assistant, who will post scores some time later. Your score will not affect your final grade, but a low score means that you should not take this course now and, if you are on the waiting list, that you cannot take it now.

1. Is $|2^{1054} - 3^{665}| < 2/3$? (Answer YES, or else NO.) _ NO _ (An even integer differs from an odd integer by at least 1 . Actually, 2^{1054} and 3^{665} differ in the last 314 of their 318 decimal digits.)

2. When the song's sentence "Everyone's talking about Heaven ain't goin' there" is rendered formally as "Everyone who is talking about Heaven is not going there," it remains ambiguous. Supply succinct unambiguous statements of both meanings using quantifiers and these predicates:

" $t(x)$ " means " Person x is talking about Heaven."
" $g(x)$ " means " Person x is going to Heaven."

One meaning: __ $\forall x\ (\ t(x) \to \neg\, g(x)\ )$ __. Another meaning: __ $\exists x\ (\ t(x) \wedge \neg\, g(x)\ )$ ___.

(These correspond to "Nobody who is talking about Heaven is going there" and "Someone who is talking about Heaven is not going there" respectively.)

3. The side-lengths of a triangle are u, v and w so ordered that $u \le v \le w$. After each of the following three inequalities write "T" if it is true, "F" if it is false, or "?" if it is not decidable from the facts stated in the previous sentence.

$w \le u + v$: _ T _. $w \le 2v$: _ T _. $v \le 2u$: _ ? _.

4. Strictly speaking, the four sentences (i), (ii), (iii), (iv) below mean four different things. Add a brief elucidation after each sentence to show that you appreciate the differences.

(i) Only birds read poetry. _ No creatures but birds read poetry. _

(ii) Birds only read poetry. _ Birds do nothing with poetry other than read it. _

(iii) Birds only read poetry. _ Birds do nothing else all the time but read poetry. _

(iv) Birds read only poetry. _ Birds don't read prose nor street signs. _

Maximum possible score: 10 points. Minimum passing score: 5 points.

Prof. W. Kahan Wed. 27 Jan. 1999

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Problem 2 illustrates a difficulty posed by the scope of negation in English. The most literal interpretation of the sentence

" Everyone who is talking about Heaven is not going there "

is the first, which denies Heaven to all who preach about it just as Moses was denied entry to the Promised Land; this is surely too harsh an interpretation for an old spiritual. The second interpretation moves the word "not" back to the beginning, either

" Not(Everyone who is talking about Heaven is going there.) "

or, colloquially,

" Not everyone who is talking about Heaven is going there."

This is the kind of difficulty that persuades good writers to avoid negatives whenever possible except perhaps in special situations like British understatement:

Bystander: "Are you hurt?" Accident victim: "Not terribly."

For similar reasons good Mathematical exposition avoids proof by contradiction whenever possible unless it is considerably shorter than the alternative.

The sentences in problem 4 are ambiguous at the outset because the English verb "read" in the third person plural is spelt the same way in the present as in the past tense. This ambiguity is innocuous enough to ignore compared with the ambiguity in the scope of "only" in American English. Attempts to put the four sentences in problem 4 into otherwise unambiguous forms with the aid of quantifiers and abstract logical expressions lead to propositions like the following:

Let logical variables m and n run over singular nouns used as the subjects or objects of verbs; this covers a rather large *Universe of Discourse*.

Let logical variable v run over verbs typically in the third person singular, present tense.

Let predicate (propositional function) $S(m, v, n)$ stand for

" Thing m does action v to thing n ."

Let predicate $b(m)$ stand for " Creature m is a bird."
Let predicate $p(n)$ stand for "Object n is poetry."
Let predicate $r(v)$ stand for "Verb v is 'reads'."

(i) Only birds read poetry. __ $\forall m \, (\, S(m, \text{reads}, \text{poetry}) \rightarrow b(m) \,)$ __.

(ii) Birds only read poetry. __ $\forall \text{bird} \, \forall v \, (\, S(\text{bird}, v, \text{poetry}) \rightarrow r(v) \,)$ __.

(iii) Birds only read poetry. __ $\forall \text{bird} \, \forall v \, \forall n \, (\, S(\text{bird}, v, n) \rightarrow (r(v) \wedge p(n)) \,)$ __.

(iv) Birds read only poetry. __ $\forall \text{bird} \, \forall n \, (\, S(\text{bird}, \text{reads}, n) \rightarrow p(n) \,)$ __.