This and other notes are available from `http://www.cs.berkeley.edu/~yozo/cs170.fa05/`

Asymptotic Notations

- Big-O (O)
  $f(n) = O(g(n))$ if there is a positive constant c such that for large enough n, $f(n) <= cg(n)$. Comparable to $<=$.
- Big-Omega (Ω)
  $f(n) = \Omega(g(n))$ if there is a positive contant c such that for large enough n, $f(n) >= cg(n)$. Comparable to $>=$.
- Big-Theta (Θ)
  $f(n) = \Theta(g(n))$ if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.
  Equivalently, $f(n) = \Theta(g(n))$ if there is positive contants c_1, c_2 such that for large enough n, $c_1 g(n) <= f(n) <= c_2 g(n)$. Comparable to $=$.
- Little-O (o)
  $f(n) = o(g(n))$ if no matter what positive c is chosen, if n gets large enough, $f(n) < cg(n)$. Comparable to $<$.
- Little-Omega (ω)
  $f(n) = \omega(g(n))$ if no matter what positive c is chosen, if n gets large enough, $f(n) > cg(n)$. Comparable to $>$.

Use of Limits

Let

$$c = \lim_{n \to \infty} \frac{f(n)}{g(n)}$$

Assuming the limit exists,

- If $c = 0, f(n) = o(g(n))$.
- If $c = \infty, f(n) = \omega(g(n))$.
- Otherwise, $f(n) = \Theta(g(n))$.

Questions

1. Show that $100n^3 + 3n^2 + 1 = O(n^3)$.
2. Show that $\log n = o(\sqrt{n})$.
3. Is there a positive function $f(n)$ such that neither $f(n) = O(n)$ nor $f(n) = \Omega(n)$ is true?
4. Show why the above “Use of Limits” holds.
5. Compare $\log n$ and $n / \log n$.
6. Compare n^n and $n!$.

7. Is $\log(n^n) = \Theta(\log(n!))$?

8. Show that

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \Theta(\log n)$$