

This and other notes are available from <http://www.cs.berkeley.edu/~yozo/cs170.fa05/>

Shortest Paths using Linear Programming

Let G be a graph with a starting vertex s . We want d_v denote the (shortest) distance from s to v . Let $\delta(s, v)$ be the true shortest distance from s to v . We can formulate the shortest path problem as follows:

$$\begin{aligned} &\text{maximize } d_t \text{ subject to} \\ & d_v \leq d_u + wt(u, v) \quad \text{for all edges } e = (u, v) \\ & d_s = 0 \\ & d_v \geq 0 \quad \text{for all nodes } v \end{aligned}$$

- The constraints imply $d_v \leq \text{length}(\mathcal{P})$, where \mathcal{P} is any path from s to v . Why? (Note: we wanted d_v to be the shortest distance from s to v , but it doesn't necessarily mean this will be true.)
- Does the above mean $d_v \leq \delta(s, v)$ for all v ?
- The maximization of d_t will make $d_t = \delta(s, t)$. Why?
- Is it true that $d_v = \delta(s, v)$ for all v ?

Max-Flow as Linear Program

Let f_{uv} be the flow on the edge $e = (u, v)$. What constraints are there? (There should be $|E| + |V| - 2$ constraints.)

Duality

Minimization of $c^T x$ subject to $Ax \geq b$ and $x \geq 0$ is equivalent to maximization of $b^T y$ subject to $A^T y \leq c$ with $y \geq 0$.

Max Flows

- Matching. Consider n car engines x_1, \dots, x_n to be matched with m cars y_1, \dots, y_m . Only certain engine-car pairs are compatible; they are indicated by existence of edges $e_{ij} = (x_i, y_j)$. Find the maximum number of engine-car matching.
- Multiple sources / sinks.
- Let $G = (V, E)$ be a directed graph with designated vertices s and t . Find the maximal number of edge-disjoint paths from s to t . (Edge-disjoint paths can't share edges, but they can share vertices).