

This and other notes are available from <http://www.cs.berkeley.edu/~yozo/cs170.fa05/>

### Depth First Search

- Edge Classification

In DFS, any edge  $e = (u, v)$  is classified as one of following:

- Tree Edge

Vertex  $v$  was first seen through this edge.

$$pre[u] < pre[v] < post[v] < post[u].$$

- Back Edge

Discovered at  $u$  going back up to the parent vertex  $v$  which in process of being explored.

$$pre[v] < pre[u] < post[u] < post[v].$$

- Forward Edge

Discovered at  $u$  going down to a descendent that exploration came back from.

$$pre[u] < pre[v] < post[v] < post[u].$$

- Cross Edge

Connects two branches of exploration.

$$pre[v] < post[v] < pre[u] < post[u].$$

- Parenthesis Property In DFS, if  $u, v$  are any vertices, then the intervals  $(pre[u], post[u])$  and  $(pre[v], post[v])$  are either disjoint, or one entirely contains the other.

- Applications

- Topological Sort

sort vertices in decreasing order of post-visit number.

- Strongly Connected Components

run DFS on  $G^R$ , run DFS on  $G$  considering the vertices in decreasing order of post-visit number.

- Cycle Detection

detect back edge

### Questions

1. In a clique of size 7, how many edges are there? How many triangles? Generalize.
2. Which of four types of edge can occur in a DFS of an *undirected* graph?
3. Is a topological sort of a graph  $G$  unique? Why or why not?
4. Prove (formally) that topological sort algorithm given in class works.
5. Give a convincing argument why the parenthesis property might hold.
6. Explain how DFS and BFS are quite similar; one uses queue instead of stack.