This and other notes are available from `http://www.cs.berkeley.edu/~yozo/cs170.fa05/`

Bellman-Ford

If the shortest path to node v is k edges long, then $d[v] = \delta(v)$ after the k-th iteration of Bellman-Ford (why?).

Minimum Spanning Trees

- Cut Property.
  The smallest edge between $S \subseteq V$ and $V - S$ must be in a MST.
- Exchange Property.

  Let T and T' be any spanning trees of G. Then given any edge $e \in T - T'$, there exists another edge $e' \in T' - T$ such that $T' \cup \{e\} - \{e'\}$ is also a spanning tree.

  Note that the exchange property allows us to move from any spanning tree T' to a MST T in a sequence of exchange steps, where each exchange does not increase the size of the spanning tree (why?).
- Prim's Algorithm
  We grow S one by one, adding the smallest edge protuding out of S.
- Kruskal's Algorithm
  We keep adding smallest edge going between two components.

Things to Review

- Asymptotic Notation (O, o, Θ, Ω, ω).
- Divide-and-conquer, recurrences, master's theorem.
- Depth First Search
  Cycle Detection, Topological Sort, Strongly Connected Components
- Shortest Paths
  Breadth First Search, Dijkstra, Bellman-Ford
- Minimum Spanning Trees
  Cut Property, Exchange Property, Kruskal, Prim

Questions

1. Is minimum spanning tree of a graph G unique? Give examples. What if all the edge weights are unique? Prove your results.
2. Give an efficient algorithm to compute the maximum spanning tree.
3. In the lecture notes, there is an assertion that if any two of the following is true about a graph G, then G is a tree.
   - $G(V, E)$ is connected.
   - $G(V, E)$ is acyclic.

- $|E| = |V| - 1$.

Prove it.

4. Prove the cut property.
5. Prove the exchange property.
6. Suppose that we add 1 to all the edges in G. How will this affect the minimum spanning tree? Shortest paths?