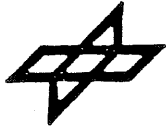


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Outline of a Computational Approach to Meaning and Knowledge Representation Based on the Concept of a Generalized Assignment Statement*

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1. Introduction

The concept of an assignment statement plays a central role in programming languages. Could it play a comparable role in the representation of knowledge expressed in a natural language? In our paper, we generalize the concept of an assignment statement in a way that makes it a convenient point of departure for representing the meaning of propositions in a natural language. Furthermore, it can be shown -- though we do not stress this issue in the present paper -- that the concept of a generalized assignment statement provides an effective computational framework for a system of inference with propositions expressed in a natural language. In some ways, this system is simpler and more direct than predicate-logic-based systems in which it is the concept of a logical form -- rather than a generalized assignment statement -- that plays a central role [7,16,23,24,25,26,28,30,31].

The approach described in the present paper may be viewed as an evolution of our earlier work on test-score semantics and canonical forms [36,38,41]. In test-score semantics, a proposition, p , is viewed as a collection of elastic constraints, and its meaning is represented as a procedure which tests, scores, and aggregates the constraints associated with p , yielding a vector test score which serves as a measure of compatibility between p and what is referred to as an *explanatory database*. The main advantage of test-score semantics over the classical approaches to meaning representation such as truth-conditional semantics, possible-world semantics and model-theoretic semantics [3,8,17,20,21,30,31], lies in its greater expressive power and, in particular, its ability to deal with fuzzy predicates such as *young*, *intelligent*, *near*, etc. [2,5,11,18,22,32,36,44]; fuzzy quantifiers exemplified by *most*, *several*, *few*, *often*, *usually*, etc. [10,33,40]; predicate modifiers such as *very*, *more or less*, *quite*, *extremely*, etc. [35,44]; and fuzzy truth-values exemplified by *quite true*, *almost true*, and *mostly false* [36].

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The concept of a generalized assignment statement serves to place in a sharper focus the representation of a proposition in a natural language as a collection of elastic constraints. More specifically, in its generic form, the generalized assignment statement may be expressed as

$$X \text{ is } r \Omega, \quad (1.1)$$

where X is the constrained variable; Ω is the constraining object, usually an n -ary predicate; and $\text{is } r$ is a copula in which r is a variable which defines the role of Ω in relation to X . The usual values of r are: d , standing for *disjunctive*; c , standing for *conjunctive*; p , standing for *probabilistic*; g , standing for *granular*; and h , standing for *hybrid*. Since in most cases the value of r is d , it is convenient to adopt the convention that $\text{is } d$ may be written more simply as is .

In (1.1), the generalized assignment statement is unconditioned. More generally, the statement may be *conditioned*, in which case it may be expressed as

$$X \text{ is } r_1 \Omega_1 \text{ if } Z \text{ is } r_2 \Omega_2, \quad (1.2)$$

in which Z is a conditioning variable; Ω_2 is an object which constrains Z ; and r_1 and r_2 are variables which define the roles of Ω_1 and Ω_2 in relation to X and Z , respectively. In general, both X and Z may be vector-valued.

Disjunctive and Conjunctive Constraints¹

As a simple illustration of a disjunctive constraint, if X is a variable which takes values in a universe of discourse U and A is a subset, A , of U , then the generalized assignment statement

$$X \text{ is } A \quad (1.3)$$

signifies that the value of X is one of the elements of A . In this sense, A may be interpreted as the *possibility distribution* of X , that is, the set of its possible values [11,36,39].

More concretely, consider the proposition

p : *Mary left home sometime between four and five in the afternoon.*

In this case, if X is taken to be the time at which Mary left home, the meaning of p may be represented as the generalized assignment statement

$$X \text{ is } d [4pm, 5pm], \quad (1.4)$$

¹ The discussion of disjunctive and conjunctive constraints in the present paper is based on earlier discussions in [36,38]. Recent results may be found in [34].

or more simply, as

$$X \text{ is } [4pm, 5pm],$$

in which the interval $[4pm, 5pm]$ plays the role of a unary predicate.

As an illustration of a conjunctive constraint, consider the proposition

p: Mary was at home from four to five in the afternoon.

In this case, if X is taken to be the time at which Mary was at home, the meaning of p may be represented as:

$$X \text{ is } [4pm, 5pm]. \quad (1.5)$$

Note that in this case X takes *all* values in the interval $[4pm, 5pm]$.

The assignment statements (1.4) and (1.5) differ from conventional assignment statements in that the assignment is set-valued rather than point-valued. Furthermore, although the assigned sets are identical in (1.4) and (1.5), they play different roles in relation to X . The possibility that the same constraining object may constrain X in different ways is the principal motivating reason for employing in (1.1) a copula of the form *isr* in which the variable r specifies the role of Ω in relation to X .

In the examples considered so far, the constraint induced by Ω is inelastic in the sense that there are only two possibilities: either the constraint is satisfied or it is not, which is characteristic of constraints associated with assignment statements in programming languages. In the case of natural languages, however, the constraints are usually elastic rather than inelastic, which implies that Ω is a fuzzy predicate. As a simple example, in the case of the proposition

p: Mary is young

the constrained variable, X , is the age of Mary, and the predicate *young* may be interpreted as an elastic constraint on X characterized by the function $\mu_{\text{young}}: [0, 100] \rightarrow [0, 1]$, which associates with each numerical value, u , of the variable *Age* the degree to which u fits the definition of *young* in the context in which p is asserted. In this sense, $1 - \mu_{\text{young}}(u)$ may be interpreted as the degree to which the predicate *young* must be stretched to fit u .

Probabilistic Constraints

As was alluded to already, a proposition p may have different generalized assignment statement representations depending on the intended meaning of p . For example, the proposition

$$p: \text{Madeleine is tall} \quad (1.6)$$

may be represented as a disjunctive statement

$$X \text{ is } TALL, \quad (1.7)$$

in which $X \triangleq \text{Height}(\text{Madeleine})$ and $TALL$ is a unary fuzzy relation which is the denotation of the fuzzy predicate *tall*.² The fuzzy relation $TALL$ is characterized by its membership function μ_{TALL} , which associates with each numerical value of height, h , the degree, $\mu_{TALL}(h)$, to which h fits the intended meaning of *tall*. Equivalently, $TALL$ may be interpreted as the possibility distribution, Π_X , of X . In this interpretation, (1.7) may be represented as

$$\Pi_X = TALL, \quad (1.8)$$

with the understanding that the possibility that X can take h as a value is given by

$$\pi_X(h) \triangleq \text{Poss} [X = h] = \mu_{TALL}(h), \quad (1.9)$$

where π_X represents the possibility distribution function of X .

Alternatively, the proposition *Madeleine is tall* may be interpreted as a characterization of the probability distribution of the variable $\text{Height}(\text{Madeleine})$. If this is the intended meaning of (1.6), then the corresponding generalized assignment statement would be probabilistic, i.e.,

$$X \text{ is}_p TALL, \quad (1.10)$$

in which $r = p$ and $TALL$ is a probability distribution. Thus, if P_X is the probability distribution of X , then (1.10) may be represented as

$$P_X = TALL \quad (1.11)$$

It should be noted that in the absence of a specification of the value of the copula variable r , the proposition

$$p: \text{Madeleine is tall}$$

may be interpreted as a possibilistic constraint on $X \triangleq \text{Height}(\text{Madeleine})$, as in (1.7), or as a probabilistic constraint, as in (1.10). We shall assume that, unless it is specifically stated that the intended interpretation of a proposition, p , is probabilistic or conjunctive, p should be interpreted as a possibilistic, i.e., disjunctive constraint. This understanding reflects the assumption that in natural languages the constraints implicit in propositions are preponderantly possibilistic in nature.

² Here and in the sequel, denotations of predicates are expressed in uppercase symbols. The symbol \triangleq stands for *is defined to be*.

A related point that should be noted is that in the possibilistic interpretation of (1.6), the value of $\pi_X(h)$ or, equivalently, $\mu_{TALL}(h)$, may be interpreted as the conditional probability of the truth of the proposition *Madeleine is tall* for a given h . In the context of a voting model, this is equivalent to viewing $\mu_{TALL}(h)$ as the proportion of voters who would vote that *Madeleine is tall* given that her height is h [13,14]. Although these interpretations are of help in developing a better understanding of the properties of the membership function, it is simplest to regard $\mu_{TALL}(h)$ as the degree to which h fits the predicate *tall* in a given context, or, equivalently, as $1 - \sigma$, where σ is the degree to which the predicate *tall* must be stretched to fit h .

Granular Constraints

In the case of a granular constraint, the generalized assignment statement assumes the form

$$X \text{ is } G \quad (1.12)$$

where X is an n -ary variable $X = (X_1, \dots, X_n)$, and G is a *granular distribution* expressed as

$$G = \{(p_1, G_1), \dots, (p_k, G_k)\} \quad (1.13)$$

in which p_1, \dots, p_k are positive numbers in the interval $[0,1]$ which add up to unity,³ and the $G_j, j=1, \dots, k$, are distinct fuzzy subsets of a universe of discourse U .

The generalized assignment statement (1.12) may be interpreted as a summary of n possibilistic assignment statements, each of which involves a component of X , i.e.,

$$X_1 \text{ is } G_{j_1} \quad (1.14)$$

...

$$X_n \text{ is } G_{j_n} .$$

in which each $G_{j_s}, s = 1, \dots, k$, is one of the G_j . In this collection of statements, p_j is the proportion of X 's which are G_j .

As an illustration, consider the following proposition

p: There are twenty residents in an apartment house; seven are old, five are young and the rest are middle-aged. (1.15)

In this case, X_i is the age of i th resident, $i = 1, \dots, 20$; $n = 20$; $k = 3$; $G_1 \triangleq \text{OLD}$; $G_2 \triangleq \text{YOUNG}$; $G_3 \triangleq \text{MIDDLE AGED}$; $p_1 = 7/20$; $p_2 = 5/20$; and $p_3 = 8/20$.

³ A more detailed discussion of the concept of a granular constraint and its role in the Dempster-Shafer theory of evidence may be found in [37].

Hybrid Constraints

A hybrid constraint is associated with a generalized assignment statement of the form

$$X \text{ ish } \Omega, \quad (1.16)$$

and may be viewed as the result of combination of two or more generalized assignment statements of different types, e.g.,

$$X \text{ isr1 } \Omega_1$$

$$\underline{X \text{ isr2 } \Omega_2}$$

$$X \text{ ish } \Omega.$$

An important special case of a hybrid constraint is associated with the concept of a hybrid number [19]. In this case, the constraint on X is characterized by two generalized assignment statements of the form

$$Y \text{ is } A \quad (1.17)$$

$$Z \text{ is } P$$

and the relation

$$X = Y + Z.$$

in which A and P are, respectively, possibility and probability distributions, and X is defined to be the sum of Y and Z . In terms of A and P , the constraining object Ω in (1.16) may be viewed equivalently as a *probabilistic set* [15], a random fuzzy set [14], or a fuzzy random variable [27].

2. Meaning Representation

As was stated already, the basic idea underlying test-score semantics is that a proposition in a natural language may be interpreted as a collection of elastic constraints. Thus, by expressing the meaning of a proposition, p , in the form of a generalized assignment statement, we are, in effect, answering two basic questions: (a) What is the constrained variable X in p ; and (b) What is the constraint, Ω , to which X is subjected?

In more concrete terms, the process of representing the meaning of a proposition, p , in the form of a generalized assignment statement, $X \text{ isr } A$, involves three basic steps.⁴

1. Constructing a collection of relations $\{R_1, \dots, R_N\}$ in terms of which the meaning of p is to be represented. The meaning of each of these relations is assumed to be known, and each relation is assumed to be characterized by its name, the names of its attributes and the domain of each attribute. For our purposes, it is convenient to

⁴ For simplicity, our discussion of these steps is limited to the possibilistic case.

refer to the collection $\{R_1, \dots, R_n\}$ as an *explanatory database* or *ED* for short, and to regard each relation as an elastic constraint on the values of its attributes. It should be noted that the concept of an explanatory database is related, but is not identical, to that of a collection of possible worlds [8,17,29,21,31].

2. Identifying the variable X which is constrained by p and constructing a defining procedure which computes X for a given explanatory database.
3. Constructing a procedure which computes the constraint A as a function of *ED*.

To illustrate this process, consider the proposition

p : *Over the past few years Naomi earned far more than all of her close friends put together.*

To represent the meaning of this proposition, assume that the explanatory database consists of the following relations (+ should be read as *and*):

$$\begin{aligned} ED = & INCOME [Name; Amount; Year] + & (2.1) \\ & FRIEND [Name1; Name2; \mu] + \\ & FEW [Number; \mu] + \\ & FAR.MORE [Income1; Income2; \mu]. \end{aligned}$$

In this database, the relation *INCOME* associates with each $Name_j$, $j = 1, \dots, n$, $Name_j$'s income in year $Year_i$, $i = 1, 2, 3, \dots$, counting backward from the present; in *FRIEND*, μ is the degree to which $Name_1$ is a friend of $Name_2$; in *FEW*, μ is the degree to which the value of the attribute *Number* fits the definition of *few*; and in *FAR.MORE*, μ is the degree to which $Income_1$ is far more than $Income_2$.

Next, we have to construct a procedure for computing the constrained variable X . Assume that X is the total income of Naomi over the past few years. Then, the following procedure will compute X .

1. Find Naomi's income, IN_i , in $Year_i$, $i = 1, 2, 3, \dots$, counting backward from present. In symbols,

$$IN_i = \text{Amount } INCOME[Name = Naomi; Year = Year_i], \quad (2.2)$$

which signifies that $Name$ is bound to Naomi, $Year$ to $Year_i$, and the resulting relation is projected on the domain of the attribute *Amount*, yielding the value of *Amount* corresponding to the values assigned to the attributes *Name* and *Year*.

2. Test the constraint induced by *FEW*:

$$\mu_i = \mu_{FEW}[Year = Year_i], \quad (2.3)$$

which signifies that the variable $Year$ is bound to $Year_i$, and the corresponding value of μ is read by projecting on the domain of μ .

3. Compute Naomi's total income, X , during the past few years:

$$X = \sum_i \mu_i N_i \quad (2.4)$$

in which the μ_i plays the role of weighting coefficients. Thus, we are tacitly assuming that the total income earned by Naomi during a fuzzily specified interval of time is obtained by (a) weighting Naomi's income in year $Year_i$, by the degree to which $Year_i$ satisfies the constraint induced by FEW , and (b) summing the weighted incomes.

The last step in the meaning representation process involves the computation of A . In words, A may be expressed as *far more than the combined income of Naomi's close friends over the past few years*. The expression for A is yielded by the following procedure.

1. Compute the total income of each $Name_i$, (other than Naomi) during the past few years:

$$TIName_i = \sum_j \mu_j IName_{ij} \quad (2.5)$$

where $IName_{ij}$ is the income of $Name_i$ in $Year_j$.

2. Find the fuzzy set of close friends of Naomi by intensifying the relation $FRIEND$ [35]:

$$CF = CLOSE.FRIEND = {}^2FRIEND \quad (2.6)$$

which implies that

$$\mu_{CF}(Name_i) = (\mu_{FRIEND}(Name = Name_i))^2,$$

where the expression

$$\mu_{FRIEND}(Name = Name_i)$$

represents $\mu_{FRIEND}(Name_i)$, that is, the grade of membership of $Name_i$ in the set of Naomi's friends.

3. Compute the combined income of Naomi's close friends:

$$CI = \sum_i \mu_{CF}(Name_i) TIName_i \quad (2.7)$$

which implies that in computing the combined income, the total income of $Name_i$ is weighted with the degree to which $Name_i$ is a close friend of Naomi.

4. The desired expression for A is obtained by substituting CI for $Income2$ in $FAR.MORE$ and projecting the result on $Income1$ and μ . Thus

$$A = \mu_{Income1} FAR.MORE [Income2 = CI] \quad (2.8)$$

In summary, the meaning of p may be represented as the possibilistic assignment statement (1.3) in which the constrained variable, X , is given by (2.4), and the elastic constraint on X is expressed by (2.8). In essence, the possibilistic assignment statement (1.3) defines the possibility distribution of X given p . What this means is that A , as ex-

pressed by (1.8), associates with each numerical value of *Income1*, the possibility that it could be far more than the combined income of Naomi's close friends over the past few years.

The same basic technique may be applied to the representation of the meaning of a wide variety of propositions in a natural language. In the following, we present in a summarized form a few representative examples.

Example 1.

$$p: \text{Richard is blond} . \quad (2.9)$$

In this case

$$p \rightarrow \text{Color(Hair(Richard)) is BLOND} , \quad (2.10)$$

where \rightarrow stands for *translates into*.

Example 2.

$$p: \text{Brian is much taller than Mildred} . \quad (2.11)$$

Here X is a binary variable (X_1, X_2) whose components are

$$X_1 = \text{Height(Brian)}$$

and

$$X_2 = \text{Height(Mildred)} .$$

The elastic constraint on $X = (X_1, X_2)$ is characterized by the fuzzy relation *MUCH.TALLER*. Thus,

$$p \rightarrow (\text{Height(Brian)}, \text{Height(Mildred)}) \text{ is MUCH.TALLER}$$

is the possibilistic assignment statement which represents the meaning of (2.11).

Example 3.

$$p: \text{most Swedes are blond} . \quad (2.12)$$

In this case, the constrained variable X is the proportion of blond Swedes among the Swedes. More specifically,

$$X = \Sigma \text{Count(BLOND/SWEDE)} , \quad (2.13)$$

where the right-hand member expresses the *relative sigma-count* [40] of blond Swedes among the Swedes. Thus, if the individuals in a sample population in Sweden are labeled *Name1*, ..., *Name_n*, then

$$\Sigma \text{Count(BLOND/SWEDE)} = \frac{\Sigma_i \mu_{\text{BLOND}}(\text{Name}_i) \wedge \mu_{\text{SWEDE}}(\text{Name}_i)}{\Sigma_i \mu_{\text{SWEDE}}(\text{Name}_i)} \quad (2.14)$$

in which $\mu_{BLOND}(Name_i)$ and $\mu_{SWEDE}(Name_i)$ represent, respectively, the degrees to which $Name_i, i = 1, \dots, n$, is blond and Swedish, and the conjunctive connective \wedge yields the minimum of its arguments.

The elastic constraint on X is characterized by the possibility distribution of the fuzzy quantifier *most*, which is a fuzzy number *MOST*. From (2.13) and (2.14), it follows that the possibilistic assignment statement which represents the meaning of (2.12) may be expressed as

$$p \rightarrow \Sigma \text{Count}(BLOND/SWEDE) \text{ is } MOST, \quad (2.15)$$

in which the constrained variable is given by (2.14).

3. Inference

One of the important advantages of employing the concept of a generalized assignment statement for purposes of meaning representation is that the process of deductive retrieval from a knowledge base is greatly facilitated when the propositions in the knowledge base are represented as generalized assignment statements. This is a direct consequence of the fact that a generalized assignment statement places in evidence the variable which is constrained and the constraint to which it is subjected.

Viewed in this perspective, a knowledge base may be equated to a collection of generalized assignment statements, and a query may be interpreted as a question regarding the value of a specified variable. Equivalently, a knowledge base may be regarded as a specification of elastic constraints on a collection of knowledge base variables X_1, \dots, X_n ; the answer to a query as the induced constraint on the variable in the query; and the inference process as the computation of the induced constraint on the query variable as a function of the given constraints on the knowledge base variables. In this view, the inference process resembles this solution of a nonlinear program [42,44].

In the following, our discussion of the problem of inference will be limited in scope. More specifically, we shall restrict our attention to disjunctive (i.e., possibilistic) assignment statements, since the inference rules for conjunctive statements can readily be derived by dualization, that is, replacing \subset (is contained in) with \supset (contains), and \cap (intersection) with \cup (union). Furthermore, we shall state only the principal rules of inference and will omit proofs.

In the rules stated below, X, Y, Z, \dots , are the constrained variables and A, B, C, \dots , are the constraining possibility distributions.

Entailment principle

$$\frac{X \text{ is } A}{\frac{A \subset B}{X \text{ is } B}}$$

(3.1)

Unary conjunctive rule

$$\frac{\begin{array}{l} X \text{ is } A \\ X \text{ is } B \end{array}}{X \text{ is } A \cap B} \quad (3.2)$$

In the conclusion, $A \cap B$ denotes the intersection of A and B , which is defined by

$$\mu_{A \cap B}(u) = \mu_A(u) \wedge \mu_B(u), \quad u \in U. \quad (3.3)$$

Binary conjunctive rule

$$\frac{\begin{array}{l} X \text{ is } A \\ Y \text{ is } B \end{array}}{(X, Y) \text{ is } A \times B} \quad (3.4)$$

where $A \times B$ denotes the cartesian product of A and B , defined by

$$\mu_{A \times B}(u, v) = \mu_A(u) \wedge \mu_B(v), \quad u \in U, \quad v \in V, \quad (3.5)$$

where U and V are the domains of X and Y , respectively.

Cylindrical extension rule

$$\frac{X \text{ is } A}{(X, Y) \text{ is } A \times V} \quad (3.6)$$

where V is the domain of Y .

Projective rule

$$\frac{(X, Y) \text{ is } A}{X \text{ is } {}_X A} \quad (3.7)$$

where ${}_X A$ denotes the projection A on the domain of X . The membership function of ${}_X A$ is defined by

$$\mu(u) = \bigvee_v (\mu_A(u, v)). \quad (3.8)$$

where \bigvee_v denotes the supremum over $v \in V$.

Compositional rule

$$\frac{\begin{array}{l} X \text{ is } A \\ (X, Y) \text{ is } B \end{array}}{Y \text{ is } A \circ B} \quad (3.9)$$

where $A \circ B$ denotes the composition of A and B , defined by

$$\mu_{A \circ B}(v) = \bigvee_u \mu_A(u) \wedge \mu_B(u, v). \quad (3.10)$$

The compositional rule may be viewed as a corollary of the cylindrical extension rule, the binary conjunctive rule and the projective rule.

Extension principle

$$\frac{X \text{ is } A}{f(X) \text{ is } f(A)} \quad (3.11)$$

where f is a function from U to V , and $f(A)$ is a possibility distribution defined by

$$\mu_{f(A)}(v) = \bigvee_u \mu_A(u) \quad , \quad \text{over all } u \text{ such that } v = f(u). \quad (3.12)$$

A more general version of the extension principle which follows from (3.4) and (3.11) is

$$\frac{\begin{array}{l} X \text{ is } A \\ Y \text{ is } B \end{array}}{f(X,Y) \text{ is } f(A,B)} \quad (3.13)$$

Generalized modus ponens

$$X \text{ is } A \quad (3.14)$$

$$\frac{\text{if } X \text{ is } B \text{ then } Y \text{ is } C}{Y \text{ is } A \circ (B' \oplus C)}$$

in which B' is the complement of B and \oplus is the bounded sum, defined by

$$\mu_{B' \oplus C}(v) = 1 \vee (1 - \mu_B(v) + \mu_C(v)) \quad , \quad (3.15)$$

where $\vee = \max$. The inference rule expressed by (3.14) follows from the compositional rule of inference (3.9) and the assumption that the meaning of the conditional assignment statement which is the second premise in (3.14) is expressed by [36]

$$\text{if } X \text{ is } B \text{ then } Y \text{ is } C \rightarrow \pi_{(Y|X)}(u,v) = 1 \vee (1 - \mu_B(u) + \mu_C(v)) \quad , \quad (3.16)$$

where $\pi_{(Y|X)}$ denotes the conditional possibility distribution function of Y given X .

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