A New Frontier in Computation—Computation with Information Described in Natural Language

Lotfi A. Zadeh

Computer Science Division
Department of EECS
UC Berkeley

ICCCC’08
May 15, 2008
Romania

URL: http://www-bisc.cs.berkeley.edu
URL: http://www.cs.berkeley.edu/~zadeh/
Email: Zadeh@eecs.berkeley.edu
PREVIEW
Science deals not with reality but with models of reality. In large measure, scientific progress is driven by a quest for better models of reality.
The real world is pervaded with various forms of imprecision and uncertainty. To construct better models of reality it is essential to develop a better understanding of the remarkable human capability to converse, communicate and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information, partiality of truth and partiality of possibility.
Formalization/mechanization of this capability is the principal motivation for the development of NL-Computation—computation with information described in natural language. NL-Computation/Computing with Words opens the door to a wide-ranging enlargement of the role of natural languages in scientific theories, especially in the realms of economics, medicine, biology, law, psychology and political science.
In conventional modes of computation, the objects of computation are values of variables. In computation with information described in natural language, or NL-Computation for short, the objects of computation are not values of variables but information about the values of variables, with the added assumption that information is described in natural language.
The role model for NL-Computation is the remarkable human capability to converse, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information or partiality of truth.
MODALITIES OF VALUATION

valuation: assignment of a value to a variable
numerical: Vera is 48

linguistic: Vera is middle-aged
Computing with Words (CW): Vera is likely to be middle-aged
NL-Computation: Vera has a teenager son and a daughter in mid-twenties
world knowledge: child-bearing age ranges from about 16 to about 42.

granular

- A concept which plays a key role in NL-Computation is that of precisiation
NL-COMPUTATION—BASIC IDEA

\[ Y = f(X) \]

- given: \( X \), information about the value of \( X \) described in natural language
- given: \( f \), information about \( f \) described in natural language
- compute: \( Y \), information about the value of \( Y \) described in natural language
EXAMPLE

\[ Y = f(X) \]

If \( X \) is small then \( Y \) is small
If \( X \) is medium then \( Y \) is large
If \( X \) is large then \( Y \) is small

\[ f \]

\( S, M, L, \) are fuzzy sets

\( *f \)

\( *X: \) usually \( X \) is medium

\( ? *Y \)
EXAMPLE (balls-in-box)

- A box contains about 20 black and white balls. Most are black. There are several times as many black balls as white balls. What is the number of white balls?

EXAMPLE (chaining)

- Overeating causes obesity
- Overeating and obesity cause high blood pressure
- I overeat. What is the probability that I will develop high blood pressure?
EXAMPLE (flight delay)

- Usually most UA flights from San Francisco leave on time. What is the probability, p, that my UA flight will be delayed?

- f-valid solution: p is low
  Usually m-most are not precisiated in consequence, p is not precisiated.

- p-valid solution: p is? Usually m-most are precisiated, in consequence, p is precisiated.

- What is your answer?
EXAMPLE (AGE DIFFERENCE)

\[ Z = \text{Age}(\text{Vera}) - \text{Age}(\text{Pat}) \]

- **Age(\text{Vera}):** Vera has a son in late twenties and a daughter in late thirties
- **Age(\text{Pat}):** Pat has a daughter who is close to thirty. Pat is a dermatologist. In practice for close to 20 years
- **NL(W1):** (relevant information drawn from world knowledge) child bearing age ranges from about 16 to about 42
- **NL(W2):** age at start of practice ranges from about 20 to about 40
- **Closed (circumscribed) vs. open (uncircumscribed)**
  - **Open:** augmentation of information by drawing on world knowledge is allowed
  - **Closed:** augmentation is not allowed
A small glass jar contains about 20 balls of various sizes. Most are small. A few are large. What is the average size of balls?

Solution

Population = \((b_1, \ldots, b_N)\)

\(d_i = \text{diameter of } b_i, \ i = 1, \ldots, N\)
A glass jar contains about 20 balls. The number of balls, \( N \), is 20.

Most are small, 
\[
\frac{\mu_{\text{small}}(d_1) + \ldots + \mu_{\text{small}}(d_N)}{N} \text{ is most}
\]

A few are large, 
\[
\mu_{\text{large}}(d_1) + \ldots + \mu_{\text{large}}(d_N) \text{ is few}
\]

What is the average size of balls?

\[
\frac{d_1 + \ldots + d_N}{N} \text{ is ?A}
\]
N is *20

\[ \left( \mu_{\text{small}}(d_1) + \ldots + \mu_{\text{small}}(d_N) \right)/N \] is most

\[ \mu_{\text{large}}(d_1) + \ldots + \mu_{\text{large}}(d_N) \] is few

\[ (d_1 + \ldots + d_N)/N \] is \( \text{?A} \)

- Solution reduces to generalized constraint propagation
As stated, the problems are not well posed. To make them well posed, what is needed is precisiation.

Precisiation is a prerequisite to computation.

Understanding is a prerequisite to precisiation.

Example:

p: Use with adequate ventilation
I understand what you mean but can you be more precise?
DISAMBIGUATION AS AN INSTANCE OF PRECISION

most tall Swedes

mostly tall Swedes

most of tall Swedes

most tall Swedes

Swedes

tall Swedes

most of tall Swedes

mostly tall Swedes
NL-COMPUTATION AND FUZZY LOGIC

- NL-Computation is based on fuzzy logic. Use of fuzzy logic in NL-Computation is a necessity rather than an option.

- Understanding of the role of fuzzy logic in NL-Computation is facilitated by taking a nontraditional view of fuzzy logic. This view highlights the role of precisiation in fuzzy logic.
The cornerstones of fuzzy logic are graduation, granulation, precisiation and granular constraint.

One of the most important features of fuzzy logic is its high power of precisiation.

(cointensive mm-precisiation)

Details follow
THE CONCEPTS OF PRECISIATION AND COINTENSIVE PRECISIATION
The concept of precision has a position of centrality in scientific theories. And yet, there are some important aspects of this concept which have not been adequately treated in the literature. One such aspect relates to the distinction between precision in value (v-precision) and precision in meaning (m-precision).

The same distinction applies to imprecision, precisiation and imprecisiation.
• **p**: X is in the interval \([a, b]\). a and b are precisely defined real numbers
• **p** is v-imprecise and m-precise

• **p**: X is a Gaussian random variable with mean \(m\) and variance \(\sigma^2\). m and \(\sigma^2\) are precisely defined real numbers
• **p** is v-imprecise and m-precise

**precise value**

**precise meaning**
A proposition, predicate, query or command may be precisiated or imprecisiated. Definition is a form of \( m \)-precisiation.

**Example**

\[
\text{young} \rightarrow m\text{-precisiation} \rightarrow \text{Lily is 25} \\
\text{Lily is 25} \rightarrow v\text{-imprecisiation} \rightarrow \text{Lily is young} \\
\text{Lily is young} \rightarrow v\text{-precisiation} \rightarrow \text{Lily is 25}
\]
PRECISIATION VS. IMPRECISIATION

\[
p \xrightarrow{\text{precisiation}} p^* \xrightarrow{\text{imprecisiation}} \ast p
\]
MODALITIES OF m-PRECISIATION

- **m-precisiation**
  - **mh-precisiation**: human-oriented (mathematically well-defined)
  - **mm-precisiation**: machine-oriented

Example: bear market

**mh-precisiation**: declining stock market with expectation of further decline

**mm-precisiation**: 30 percent decline after 50 days, or a 13 percent decline after 145 days. (Robert Shuster)
CONTINUED

- Risk\textsuperscript{mh-precisiation} exposure to the chance of injury or loss
- Risk\textsuperscript{mm-precisiation} expected value of loss function
**BASIC CONCEPTS**

- **precisiation** language

- **precisiend** = model of meaning
- **intension** = attribute-based meaning
- **cointension** = measure of closeness of meanings
  = measure of goodness of model

- A **precisiend** has many **precisiands**. \( \text{Pres}(p) \) denotes the set of **precisiands** of \( p \)

**precisiation** = translation into a **precisiation language**

\[ p: \text{object of precisiation} \]

\[ \text{precisiend} \rightarrow \text{precisiation} \rightarrow \text{precisiand} \]

\[ p*: \text{result of precisiation} \]

\[ \text{cointension} \]
**PRECISIATION AND MODELING**

- **mm-precisiand** = *mathematical model*
- **mm-precisiation** = *construction of a mathematical model*

- Unless stated to the contrary, precisiation should be interpreted as **mm-precisiation**
MM-PRECISIATION OF “approximately a,” *a (MODELS OF MEANING OF *a)

Bivalent Logic

- **number**
- **interval**
- **probability**

![Graphs showing the representation of *a* in different contexts: number, interval, and probability.](image)
CONTINUED

Fuzzy Logic: Bivalent Logic + ...

fuzzy interval

fuzzy interval

fuzzy probability
v-IMPRECISIATION

- **v-imprecisiation**
  - Imperative (forced)
  - Intentional (deliberate)

**imperative:** value is not known precisely

**intentional:** value need not be known precisely

- **v-imprecisiation principle:** Precision carries a cost. If there is a tolerance for imprecision, exploit it by employing v-imprecisiation to achieve lower cost, robustness, tractability, decision-relevance and higher level of confidence. Employ mm-precisciation to achieve computability.

- **data compression and summarization are instances of v-imprecisiation**
THE FUZZY LOGIC GAMBIT

\[ p \rightarrow v\text{-imprecisiation} \rightarrow mm\text{-precisiation} \]

- reduction in cost
- achievement of computability

Fuzzy logic gambit = v-imprecisiation followed by mm-precisiation

Lily is 25 → Lily is young

LAZ 5/9/2008
THE CONCEPT OF COINTENSIVE PRECISIATION

- m-precisiation of a concept or proposition, p, is cointensive if p* is cointensive with p.

Example: bear market

We classify a bear market as a 30 percent decline after 50 days, or a 13 percent decline after 145 days. (Robert Shuster)

This definition is clearly not cointensive
EXAMPLE: IMPACT FACTOR

- \( A = \text{citations in 1992 to articles published in 1987-91} \)

- \( B = \text{articles published in 1987-91} \)

- \( C = \frac{A}{B} = \text{five-year impact factor} \)

- Is the definition of impact factor cointensive?
mm-PRECISIATION

Basic questions

a) Given a proposition, \( p \), how can \( p \) be cointesively mm-precisiated?

b) How can mm-precisian of \( p \) be treated as an object of computation/deduction?

In NL-Computation these questions are addressed through the use of fuzzy-logic-based computational semantics of natural languages.
FUZZY-LOGIC-BASED COMPUTATIONAL SEMANTICS OF NATURAL LANGUAGES

GCS

generalized-constraint-based semantics (Zadeh 2006)

TCS

test-score semantics (Zadeh 1982)

GCS: point of departure

p is a proposition

X is a variable which is explicit or implicit in p

p is interpreted as an answer to the question: What is the value of X and how is it derived?
TSS: point of departure

What is the truth value, t, of p, and how is it derived?

GCS and TSS are closely related. Underlying both is the concept of a generalized constraint. A brief outline of this concept is presented in the following.
THE CONCEPT OF A GENERALIZED CONSTRAINT AND ITS ROLE IN COMPUTATIONAL SEMANTICS OF NATURAL LANGUAGES
The concept of a generalized constraint is the centerpiece of generalized-constraint-based semantics. An outline of this concept if presented in the following.

In scientific theories, representation of constraints is generally oversimplified. Oversimplification of constraints is a necessity because existing constrained definition languages have a very limited expressive power.
The concept of a generalized constraint is intended to provide a basis for construction of a maximally expressive constraint definition language which can also serve as a meaning representation/precisiation language for natural languages.
GENERALIZED CONSTRAINT (Zadeh 1986)

- Bivalent constraint (hard, inelastic, categorical:)
  \[ X \in C \]
  constraining bivalent relation

- Generalized constraint on \( X \): \( \text{GC}(X) \)

\[ \text{GC}(X): \text{X isr R} \]

constraining non-bivalent (fuzzy) relation

index of modality (defines semantics)

constrained variable

\[ r: \in \mid = \mid \leq \mid \geq \mid \subset \mid \ldots \mid \text{blank} \mid p \mid v \mid u \mid rs \mid fg \mid ps \mid \ldots \]

bivalent

primary

- open \( \text{GC}(X): X \) is free (\( \text{GC}(X) \) is a predicate)
- closed \( \text{GC}(X): X \) is instantiated (\( \text{GC}(X) \) is a proposition)
GENERALIZED CONSTRAINT—MODALITY $r$

$X isr R$

$r: =$ equality constraint: $X = R$ is abbreviation of $X is = R$

$r: \leq$ inequality constraint: $X \leq R$

$r: \subset$ subsethood constraint: $X \subset R$

$r: \text{blank}$ possibilistic constraint; $X is R; R$ is the possibility distribution of $X$

$r: v$ veristic constraint; $X isv R; R$ is the verity distribution of $X$

$r: p$ probabilistic constraint; $X isp R; R$ is the probability distribution of $X$

Standard constraints: bivalent possibilistic, bivalent veristic and probabilistic
CONTINUED

$r$: bm  bimodal constraint; $X$ is a random variable; $R$ is a bimodal distribution

$r$: rs  random set constraint; $X$ isrs $R$; $R$ is the set-valued probability distribution of $X$

$r$: fg  fuzzy graph constraint; $X$ isfg $R$; $X$ is a function and $R$ is its fuzzy graph

$r$: u   usuality constraint; $X$ isu $R$ means usually ($X$ is $R$)

$r$: g   group constraint; $X$ isg $R$ means that $R$ constrains the attribute-values of the group
PRIMARY GENERALIZED CONSTRAINTS

- Possibilistic: X is R
- Probabilistic: X isp R
- Veristic: X isv R

- Primary constraints are formalizations of three basic perceptions: (a) perception of possibility; (b) perception of likelihood; and (c) perception of truth

- In this perspective, probability may be viewed as an attribute of perception of likelihood
EXAMPLES: POSSIBILISTIC

- Monika is young $$\rightarrow$$ Age (Monika) is young

- Monika is much younger than Maria $$\rightarrow$$ (Age (Monika), Age (Maria)) is much younger

- most Swedes are tall $$\rightarrow$$ Count (tall.Swedes/Swedes) is most
EXAMPLES: PROBABILISTIC

- $X$ is a normally distributed random variable with mean $m$ and variance $\sigma^2$ \[ X \sim N(m, \sigma^2) \]

- $X$ is a random variable taking the values $u_1, u_2, u_3$ with probabilities $p_1, p_2$ and $p_3$, respectively \[ X \sim (p_1u_1 + p_2u_2 + p_3u_3) \]
EXAMPLES: VERISTIC

- Robert is half German, quarter French and quarter Italian
  
  Ethnicity (Robert) isv (0.5|German + 0.25|French + 0.25|Italian)

- Robert resided in London from 1985 to 1990
  
  Reside (Robert, London) isv [1985, 1990]
STANDARD CONSTRAINTS

- **Bivalent possibilistic:** \( X \in C \) (crisp set)
- **Bivalent veristic:** \( \text{Ver}(p) \) is true or false
- **Probabilistic:** \( X \text{ isp } R \)
- **Standard constraints are instances of generalized constraints which underlie methods based on bivalent logic and probability theory**
GENERALIZED CONSTRAINT LANGUAGE (GCL)

- GCL is generated by combination, qualification, propagation and counterpropagation of generalized constraints

- examples of elements of GCL
  - \((X \text{ isp } R) \text{ and } (X,Y) \text{ is } S)\)
  - \((X \text{ isr } R) \text{ is unlikely} \) and \((X \text{ iss } S) \text{ is likely}\)
  - If \(X\) is \(A\) then \(Y\) is \(B\)

- the language of fuzzy if-then rules is a sublanguage of GCL
CLARIFICATION
LANGUAGE VS. LANGUAGE SYSTEM

- Language = (description system)
- Description system = (syntax, semantics)
- Language system = (description system, computation/deduction system)

Examples: Java is a language; Prolog is a language system; probability theory is a language system; fuzzy logic is a system of language systems.

- Generalized Constraint Language (GCL) is a language system.
- The rules of deduction in GCL are the rules which govern propagation and counterpropagation of generalized constraints.
The principal rule of deduction in NL-Computation is the Extension Principle (Zadeh 1965, 1975).

\[ f(X) \text{ is } A \implies g(X) \text{ is } B \]

\[ \mu_B(v) = \sup_u \mu_A(f(u)) \]

subject to

\[ v = g(u) \]
**PROTOFORMAL DEDUCTION RULE**

- **Syllogism**

  \[
  \begin{align*}
  Q_1 & \text{ A’s are B’s} \\
  Q_2 & \text{ (A&B)’s are C’s} \\
  Q_1 Q_2 & \text{ A’s are (B&C)’s}
  \end{align*}
  \]

**Example**

- Overeating causes obesity \(\xrightarrow{\text{precisiation}}\) most of those who overeat become obese
- Overeating and obesity cause high blood pressure \(\xrightarrow{\text{precisiation}}\) most of those who overeat and are obese have high blood pressure
- I overeat and am obese. The probability that I will develop high blood pressure is most\(^2\)
PROTOFORMAL DEDUCTION RULE

\[
\frac{1/n \sum \text{Count}(G[H \text{ is } R]) \text{ is } Q}{1/n \sum \text{Count}(G[H \text{ is } S]) \text{ is } T}
\]

\[
\frac{\sum_i \mu_R(h_i) \text{ is } Q}{\sum_i \mu_S(h_i) \text{ is } T}
\]

\[
\mu_T(v) = \sup_{h_1, \ldots, h_n} (\mu_Q(\sum_i \mu_R(h_i)))
\]

subject to

\[
v = \sum_i \mu_S(h_i)
\]

values of H: \( h_1, \ldots, h_n \)
Generalized-constraint-based semantics suggests a novel, powerful approach to semantics of natural languages. It is generalized-constraint-based semantics that opens the door to computation with information described in natural language.

The point of departure in generalized-constraint-based semantics is the fundamental thesis of fuzzy logic.

Information = generalized constraint

Basically, what this means is that information about a variable, X, may be viewed as a constraint on the values which X can take.
A proposition, $p$, is a carrier of information.

A consequence of the fundamental thesis is the meaning postulate.

$\text{meaning of } p = \text{generalized constraint}$

In NL-Computation, the meaning of $p$ is equated with its mm-precisiand. More specifically
MEANING POSTULATE—A RATIONALE

- A proposition, $p$, may be viewed as an answer to a question, $q$.
- A question can be expressed as: What is the value of $X$? Where $X$ is explicit or implicit in $p$.
- A generalized constraint may be interpreted as an answer to a question. From this it follows that the answer to $q$ may be expressed as a generalized constraint.

$$X \text{ isr } R$$

- In general $X$ and $R$ are implicit in $p$. In this sense, the meaning of $p$ may be expressed as a generalized constraint in which $X$ and $R$ are defined procedurally.
- Note that $X$ is a variable that is focused on but is not uniquely determined by $X$. For this reason, $X$ is referred to as a focal variable.
What should be stressed is that mm-precisiation is not the final goal. It is a preliminary to computation/deduction. The roles of mm-precisiation and computation/deduction are illustrated in the following.
PRECISIATION AND COMPUTATION/DEDUCTION—EXAMPLE

- $p$: most Swedes are tall
  $p^*$: $\Sigma \text{Count(tall.Swedes/Swedes)}$ is most

- $q$: How many are short?
  further precisiation

$X(h)$: height density function (not known)
$X(h)du$: fraction of Swedes whose height
is in $[h, h+du]$, $a \leq h \leq b$

$$\int_a^b X(h)du = 1$$
**CONTINUED**

- **fraction of tall Swedes:** $\int_a^b X(h)\mu_{tall}(h)\,dh$

- **constraint on $X(h)$**

$$\int_a^b X(h)\mu_{tall}(h)\,dh$$

is most granular value

$$\pi(X) = \mu_{most}\left(\int_a^b X(h)\mu_{tall}(h)\,dh\right)$$
CONTINUED

deduction:

\[ \int_a^b X(h) \mu_{\text{tall}}(h) dh \text{ is most given} \]

\[ \int_a^b X(h) \mu_{\text{short}}(h) dh \text{ is } ? Q \text{ needed} \]

solution:

\[ \mu_Q(v) = \sup_{X} (\mu_{\text{most}}(\int_a^b X(h) \mu_{\text{tall}}(h) dh)) \]

subject to

\[ v = \int_a^b X(h) \mu_{\text{short}}(h) dh \]

\[ \int_a^b X(h) dh = 1 \]
In a general setting, computation/deduction is governed by the Deduction Principle.

Point of departure: question, q

Data: \( D = (X_1/u_1, \ldots, X_n/u_n) \)

\( u_i \) is a generic value of \( X_i \)

\( \text{Ans}(q) \): answer to q
If we knew the values of the $X_i$, $u_1$, ..., $u_n$, we could express $\text{Ans}(q)$ as a function of the $u_i$

$$\text{Ans}(q) = g(u_1, ..., u_n) \quad u = (u_1, ..., u_n)$$

Our information about the $u_i$, $I(u_1, ..., u_n)$ is a generalized constraint on the $u_i$. The constraint is defined by its test-score function

$$f(u) = f(u_1, ..., u_n)$$
Use the extension principle

\[ \mu_{Ans(q)}(v) = \sup_u (ts(u)) \]

subject to

\[ v = g(u) \]
**EXAMPLE**

*p: Most Swedes are much taller than most Italians*

*q: What is the difference in the average height of Swedes and Italians?*

**Solution**

**Step 1. precisiation: translation of p into GCL**

- \( S = \{S_1, \ldots, S_n\} \): population of Swedes
- \( I = \{I_1, \ldots, I_n\} \): population of Italians
- \( g_i = \text{height of } S_i \), \( g = (g_1, \ldots, g_n) \)
- \( h_j = \text{height of } I_j \), \( h = (h_1, \ldots, h_n) \)

\[ \mu_{ij} = \mu_{\text{much.taller}}(g_i, h_j) = \text{degree to which } S_i \text{ is much taller than } I_j \]
\[ r_i = \frac{1}{n} \sum_j \mu_{ij} \] = Relative \( \Sigma \) Count of Italians in relation to whom \( S_i \) is much taller

\[ t_i = \mu_{\text{most}}(r_i) = \text{degree to which } S_i \text{ is much taller than most Italians} \]

\[ v = \frac{1}{m} \sum_i t_i \] = Relative \( \Sigma \) Count of Swedes who are much taller than most Italians

\[ t_s(g, h) = \mu_{\text{most}}(v) \]

\[ p \rightarrow \text{generalized constraint on } S \text{ and } I \]

\[ q: d = \frac{1}{m} \sum_i g_i - \frac{1}{n} \sum_j h_j \]
Step 2. Deduction via Extension Principle

\[ \mu_q(d) = \sup_{g,h} \text{ts}(g,h) \]

subject to

\[ d = \frac{1}{m} \sum_{i} g_i - \frac{1}{n} \sum_{j} h_j \]
EPILOGUE
THE NEED FOR PRECISIATION OF NATURAL LANGUAGE

- Natural languages are intrinsically imprecise. A prerequisite to computation with information described in natural language is mm-precisiation of meaning. Mm-precisiation of meaning is the first step in NL-Computation. The concept of generalized-constraint plays a key role in both precisiation and computation/deduction.
NL-INCAPABILITY

- NL-capability = capability to operate on information described in natural language

- Existing scientific theories are based for the most part on bivalent logic and bivalent-logic-based probability theory

- Bivalent logic and bivalent-logic-based probability theory do not have NL-capability

- For the most part, existing scientific theories do not have NL-capability
NL-CAPABILITY

- NL-Computation serves to add NL-capability to existing scientific theories and thereby opens the door to a wide ranging enlargement of the role of natural languages in scientific theories, especially in human-centric theories such as economics, linguistics, law and psychology.

- An important application of NL-Computation relates to computation with imprecise probabilities and probability distributions described in natural language.

- Generally, real-world probabilities and probability distributions are not known precisely.
A KEY OBSERVATION

- The concept of a set has a position of centrality in mathematics. A set is a special set of a fuzzy set. Generalization of the concept of a set is the point of departure in fuzzy logic and underlies its generality and power. More generally, the concept of a fuzzy set opens the door to generalization of many branches of mathematics, among them topology, algebra, functional analysis and theory of relations.
More over, it serves as a basis for upgrading or, more concretely, FL enhancement, of bivalent-logic-based scientific theories through addition of concepts and techniques drawn from fuzzy logic. In the limit, FL enhancement leads to a shift in the foundations of a scientific theory from bivalent logic to fuzzy logic.
A shift from bivalent logic to fuzzy logic results in a theory which has an enhanced capability to model reality. The enhanced capability to model reality is one of the main contributions of fuzzy logic.


CONTINUED

- From computing with numbers to computing with words --from manipulation of measurements to manipulation of perceptions, IEEE Transactions on Circuits and Systems 45, 105-119, 1999.


Factual Information About the Impact of Fuzzy Logic

PATENTS

- Number of fuzzy-logic-related patents applied for in Japan: 17,740
- Number of fuzzy-logic-related patents issued in Japan: 4,801
- Number of fuzzy-logic-related patents issued in the US: around 1,700

Number of papers in INSPEC and MathSciNet which have "fuzzy" in title:

**INSPEC - "fuzzy" in title**
- 1970-1979: 569
- 1980-1989: 2,403
- 1990-1999: 23,214
- 2000-present: 24,910
- **Total: 51,096**

**MathSciNet - "fuzzy" in title**
- 1970-1979: 443
- 1980-1989: 2,465
- 1990-1999: 5,487
- 2000-present: 6,217
- **Total: 14,612**
JOURNALS ("fuzzy" in title)

1. Fuzzy in title
2. Fuzzy Sets and Systems
3. IEEE Transactions on Fuzzy Systems
4. Fuzzy Optimization and Decision Making
5. Journal of Intelligent & Fuzzy Systems
6. Fuzzy Economic Review
10. International Review of Fuzzy Mathematics
11. Fuzzy Systems and Soft Computing