IS THERE A NEED FOR FUZZY LOGIC?

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IS THERE A NEED FOR FUZZY LOGIC?
NO, THERE IS NO NEED

- [Dennis Lindley (1987)] The only satisfactory description of uncertainty is probability. By this I mean that every uncertainty statement must be in the form of a probability; that several uncertainties must be combined using the rules of probability; and that the calculus of probabilities is adequate to handle all situations involving uncertainty...probability is the only sensible description of uncertainty and is adequate for all problems involving uncertainty.
All other methods are inadequate...anything that can be done with fuzzy logic, belief functions, upper and lower probabilities, or any other alternative to probability can better be done with probability.
“Fuzzy theory is wrong, wrong, and pernicious.” says William Kahan, a professor of computer sciences and mathematics at Cal whose Evans Hall Office is a few doors from Zadeh’s. “I can not think of any problem that could not be solved better by ordinary logic.” (Kahan, 1975)

Professor Susan Haack, a prominent logician and philosopher, commenting on the need for fuzzy logic in her book “Deviant Logic Fuzzy Logic”. “Since neither of the main arguments that are offered in its favor is acceptable, I conclude that we do not need fuzzy logic.” (Haack 1974)
IS THERE A NEED FOR FUZZY LOGIC?
YES, THERE IS

- Why? What does fuzzy logic have to offer?
- To formulate an insightful answer to this question it is expedient to look at fuzzy logic, FL, in a nontraditional perspective. In this perspective, the cornerstones of fuzzy logic are: graduation, granulation, precisiation and the concept of a generalized constraint.
PRINCIPAL CONTRIBUTIONS OF FUZZY LOGIC

FL

FL-generalization

precisiation/modelization
Fuzzy logic opens the door to generalization/upgrading of any theory, $T$, through addition to $T$ of concepts and techniques drawn from fuzzy logic. Generally, the upgraded theory, $T^*$, has enhanced problem-solving capability.

- control theory $\rightarrow$ fuzzy control theory
- probability theory $\rightarrow$ fuzzy probability theory
- measure theory $\rightarrow$ fuzzy measure theory
- arithmetic $\rightarrow$ fuzzy arithmetic
- ...

FL-GENERALIZATION
One of the principal contributions of fuzzy logic is its high power of cointensive precisiation/modelization.

Cointension of $p$ and $p^*$ = a measure of proximity of meanings/input-output relations of $p$ and $p^*$.

FL serves as a high power precisiation/modelization language system.

Language system: (description, deduction)
SIMPLE EXAMPLE OF PRECISIATION

middle-aged:

precisiation/modelization

precisiation/modelization

precisiation/modelization

middle-aged
WHAT IS FUZZY LOGIC AND WHAT DOES IT HAVE TO OFFER?
A NONTRADITIONAL VIEW
Fuzzy logic is rooted in the concept of a fuzzy set. In formally, a fuzzy set is a class with unsharp boundaries. A fuzzy set is defined by its membership function.
The principal attributes of a class are: a) boundary; and b) count/volume/measure

The mathematics of probability theory is the mathematics of measure theory

The mathematics of fuzzy set theory is the mathematics of membership functions

The mathematics of fuzzy logic, FL, is the mathematics of the union of mathematics of fuzzy set theory and the mathematics of membership functions
Science deals not with reality but with models of reality. In large measure, scientific progress is driven by a quest for better models of reality.
In the real world, fuzziness is a pervasive phenomenon.

To construct better models of reality it’s necessary to develop a better understanding of how to deal precisely with unsharpness of class boundaries. In large measure, fuzzy logic is motivated by this need.
WHAT IS FUZZY LOGIC?

- There are many misconceptions about fuzzy logic.

- Fuzzy logic is fuzzy. wrong

- Fuzzy logic is not fuzzy. Fuzzy logic is a precise logic of imprecision. correct

- Fuzzy logic, FL, competes with probability theory, PT. wrong

- Probability theory, PT, may be enriched through addition to PT of concepts and techniques drawn from fuzzy logic, FL. correct
PROBABILITY THEORY & FUZZY LOGIC?

wrong

correct

PNL: precisiated natural language
fuzzy logic is much more than a logical system

fuzzy logic (FL) has four principal facets

- **FL/L** logical (narrow sense FL)
- **FL/S** set-theoretic
- **FL/R** relational
- **FL/E** epistemic

**F**: fuzziness/ fuzzification
**G**: granularity/ granulation
**F.G**: F and G
The cornerstones of fuzzy logic are graduation, granulation, precisiation and granular constraint.

One of the most important features of fuzzy logic is its high power of \( \wedge \) precisiation.

\( \text{(cointensive \( mm \)-precisiation) \)} \)
FUZZY LOGIC—AN OBJECT OF CONTROVERSY

- “Pattern recognition analysis and fuzzy logic analysis of breath VOCs independently distinguished healthy controls from hospitalized patients with 100% sensitivity and 100% specificity.” (Dr. Michael Phillips, 2007)

- “Because of this sophisticated functionality and performance realized by fuzzy logic, the number of sale of our Omron's blood pressure meter in all over the world is over 8,000,000 per year now.” (Hiroshi Nakajima, 2007)

Arthur Geoffrion (WMSI)

I agree with your remark during the Colloquium that the present development of fuzzy sets is probably still overly restrictive for many potential applications. It is applicable when there is a standardized and perfectly accurate method of measuring degree of class membership, but it seems to be considerably less applicable otherwise, as when opinion is involved.
H.P. Edmundson (Linguistics, UCLA)

Professor Zadeh’s view that the concept of fuzzy sets seems to be needed in many disciplines has been supported by comments today from specialists in the fields of psychology, economics, and logic. In this connection I would like to point out that fuzzy sets also arise in linguistics. In particular, in the study of semantics, attempts to formulate satisfactorily the notion of a semantic space using crisp sets has essentially failed.
As a consequence, the modeling of meaning as a set of senses or the modeling of synonymy in terms of equivalence classes has proved difficult to justify either theoretically or empirically. It seems likely that the concept of fuzzy set will provide a way to account for what has been called a “semantic space” and lead to a suitable metric or pseudometric.
Similarly, it also may lead to a satisfactory way to replace the strict dichotomy of sentences as grammatical or ungrammatical, by a more natural concept involving grade of membership.
FUZZY LOGIC—PRINCIPAL RATIONALES

Rationale A: precise modeling of imprecision
Rationale B: exploiting tolerance for imprecision through deliberate sacrifice of precision followed by precise modeling of imprecision. (Fuzzy Logic Gambit)

Industrial control
Train control
Traffic control
Automobile transmissions
Automobile climate control
Automobile body painting control
Automobile engine control
Paper manufacturing
Steel manufacturing
...

Economics
Operation research
Decision analysis
Fraud detection
Pattern classification
Chemistry
Mathematics
Medicine
Biomedical instrumentation
...

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Imperfect information = information which is imprecise and/or uncertain and/or incomplete and/or partially true.

Rationale A: fuzzy logic is designed to deal with ip-systems.

Rationale B: if a p-system is associated with a tolerance for imprecision, then the tolerance for imprecision may be exploited through deliberate imprecisiation which reduces cost. Imprecisiation is followed by precisiation of meaning (fuzzy logic gambit).
In one form or another, precisiation of meaning has always played an important role in science. Mathematics is a quintessential example of what may be called a meaning precisiation language. Precisiation of meaning has direct relevance to mechanization of natural language understanding. For this reason, precisiation of meaning is an issue that is certain to grow in visibility and importance as we move further into the age of machine intelligence and automated reasoning. The concept of precisiation plays a pivotal role in the nontraditional view of fuzzy logic.
Semantic imprecision of natural languages is a very basic characteristic—a characteristic which is rooted in imprecision of perceptions. Basically, a natural language is a system for describing perceptions. Perceptions are imprecise. Imprecision of perceptions entails semantic imprecision of natural languages.
SEMANTIC IMPRECISION (EXPLICIT)

EXAMPLES

WORDS/CONCEPTS
- Recession
- Slow
- Very slow
- Honesty
- Arthritis
- High blood pressure
- Civil war
- Cluster

PROPOSITIONS
- It is likely to be warm tomorrow.
- A box contains about 20 balls of various sizes. Most are small. There are many more small balls than large balls.
CONTINUED

EXAMPLES

PROPOSITIONS

- Usually most UA flights leave on time. Rarely most are delayed.
- It is very unlikely that there will be a significant decrease in the price of oil in the near future.

COMMANDS

- Slow down
- Slow down if foggy
- Park the car
SEMANTIC IMPRECISION (IMPLICIT)

EXAMPLES

- Speed limit is 65 mph
- Checkout time is 1 pm
NECESSITY OF IMPRECISION

- Can you explain to me the meaning of “Speed limit is 65 mph?”
- No imprecise numbers and no probabilities are allowed
- Imprecise numbers are allowed. No probabilities are allowed.
- Imprecise numbers are allowed. Precise probabilities are allowed.
- Imprecise numbers are allowed. Imprecise probabilities are allowed.
NECESSITY OF IMPRECISION

- Can you precisiate the meaning of “arthritis”? 
- Can you precisiate the meaning of “recession”? 
- Can you precisiate the meaning of “beyond reasonable doubt”? 
- Can you precisiate the meaning of “causality”? 

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Imprecision of meaning = elasticity of meaning

Elasticity of meaning = fuzziness of meaning

Example: middle-aged
Traditional approaches to semantics of natural languages, among them truth-conditional semantics, possible-world semantics and Montague semantics, do not address the issue of semantic imprecision of natural languages. The issue is not addressed because the conceptual framework of traditional approaches is not the right framework for dealing with semantic imprecision. In traditional approaches, elasticity of meaning is not dealt with.
There is a need for new direction. It is this need that motivates generalized-constraint-based semantics of natural languages, or GCS for short. As an issue, semantic imprecision has a position of centrality in GCS.

The point of departure in GCS is the concept of precisiation of meaning—a concept which goes beyond the familiar concept of representation of meaning.
REPRESENTATION VS. PRECISIATION

semantics

meaning
representation

meaning
precisiation

r-semantics
traditional

p-semantics
nontraditional

Lily is young representation Age(Lily) is young presentation

\( \mu \)

young

Age

1

0
most Swedes are tall  \( \text{representation} \)

\[ \text{Count}(\text{tall}.\text{Swedes}/\text{Swedes}) \text{ is most.} \]

\[ \text{Count}(\text{tall}.\text{Swedes}/\text{Swedes}) \text{ precisiation} \]

\[ \sum \text{Count}(\text{tall}.\text{Swedes}/\text{Swedes}) \]
THE CONCEPTS OF PRECISION AND COINTENSIVE PRECISION
The concept of precision has a position of centrality in scientific theories. And yet, there are some important aspects of this concept which have not been adequately treated in the literature. One such aspect relates to the distinction between precision of value (v-precision) and precision of meaning (m-precision).

The same distinction applies to imprecision, precisiation and imprecisiation.
• \( p: X \text{ is in the interval } [a, b]. \ a \text{ and } b \text{ are precisely defined real numbers} \\
• \( p \) is \( v \)-imprecise and \( m \)-precise

• \( p: X \text{ is a Gaussian random variable with mean } m \text{ and variance } \sigma^2. \ m \text{ and } \sigma^2 \text{ are precisely defined real numbers} \\
• \( p \) is \( v \)-imprecise and \( m \)-precise
A proposition, predicate, query or command may be precisiated or imprecisiated.

Examples:

- **m-precisiation**: Lily is 25
- **v-imprecisiation**: Lily is young
- **v-precisiation**: Lily is 25

Graphically:

- Lily is 25 → m-precisiation → Lily is young
- Lily is young → v-imprecisiation → Lily is 25
PRECISION VS. IMPRECISION

- Lily is young → **precisiation** → Age(Lily is young),
- Lily is 25 → **imprecisiation** → Lily is young
MODALITIES OF M-PRECISIATION

- **m-precisiation**
  - **mh-precisiation**: declining stock market with expectation of further decline
  - **mm-precisiation**: 30 percent decline after 50 days, or a 13 percent decline after 145 days. (Robert Shuster)
EXAMPLES

Recession (mh-precisiation):
- A period of an economic contraction, sometimes limited in scope or duration.

Recession (mm-precisiation):
- A recession is a decline in a country's gross domestic product (GDP), or negative real economic growth, for two or more successive quarters of a year.

- Risk \(_{\text{mh-precisiation}}\) exposure to the chance of injury or loss
- Risk \(_{\text{mm-precisiation}}\) expected value of loss function
BASIC CONCEPTS

precisiend $\rightarrow$ precisiation $\rightarrow$ precisiation language $\rightarrow$ precisiation

cointension

- \textit{precisiand} = model of meaning
- \textit{intension} = attribute-based meaning
- \textit{cointension} = measure of closeness of meanings
  $= \text{measure of goodness of model}$
- A \textit{precisiend} has many \textit{precisiands}.

\textit{precisiation} = translation into a \textit{precisiation language}

$p: \text{object of precisiation}$

$p^*: \text{result of precisiation}$
It is a common practice to ignore imprecision, treating what is imprecise as if it were precise.
Fuzzy Logic: Bivalent Logic + ...

Fuzzy logic has a much higher expressive power than bivalent logic.
GOODNESS OF MODEL OF MEANING

goodness of model = (cointension, computational complexity)

*a: approximately a

two numbers

four numbers

one number

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**PRECISIATION IN COMMUNICATION**

- **HHC**
  - *human-human communication*
  - *mm-precisiation is desirable but not mandatory*
  - *Humans can understand unprecisiated natural language. Machines cannot.*

- **HMC**
  - *human-machine communication*
  - *mm-precisiation is mandatory*

- **mh-precisiation** → **mm-precisiation**

*scientific progress*
Recipient: I understand what you sent, but could you precisiate what you mean, using ... (restrictions)?

Sender: (a) I will be pleased to do so (s-precisiation)  
(b) sorry, it is your problem (r-precisiation)
In human-machine communication, precisiation is a necessity. Where does precisiation take place?

- In mechanization of natural language understanding, the precisiator is the machine.
- In most applications of fuzzy logic, the precisiator is the human. In this case, context-dependence is not a problem. As a consequence, precisiation is a much simpler function.
HUMAN-HUMAN COMMUNICATION (HHC)

- **mm-precisiation is desirable but not mandatory.**
- **mm-precisiation in HHC is a major application area for generalized-constraint-based deductive semantics.**
- **Reformulation of bivalent-logic-based definitions of scientific concepts, associating Richter-like scales with concepts which are traditionally defined as bivalent concepts but in reality are fuzzy concepts.**
- **Examples: recession, civil war, stability, arthritis, boldness, etc.**
HONDA FUZZY LOGIC TRANSMISSION

Control Rules:

1. If (speed is low) and (shift is high) then (-3)
2. If (speed is high) and (shift is low) then (+3)
3. If (throt is low) and (speed is high) then (+3)
4. If (throt is low) and (speed is low) then (+1)
5. If (throt is high) and (speed is high) then (-1)
6. If (throt is high) and (speed is low) then (-3)
EXAMPLE OF r-PRECISIATION

- How old is Vera?
- Vera has a son in mid-twenties. How old is Vera?
- Vera has a son in mid-twenties and a daughter in mid-thirties. How old is Vera?
- Vera has a son in mid-twenties and a daughter in mid-thirties. The child-bearing age ranges from approximately 16 to approximately 42. How old is Vera?
- What is the probability distribution of Vera’s age?
v-IMPRECISIATION

**v-imprecisiation**

- *Imperative (forced)*
- *Intentional (deliberate)*

**imperative:** value is not known precisely

**intentional:** value need not be known precisely

- data compression and summarization are instances of v-imprecisiation
**The Fuzzy Logic Gambit**

Fuzzy logic gambit = v-imprecisiation followed by mm-precisiation

- Lily is 25
- Lily is young

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achievement of computability
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reduction in cost
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Most applications of fuzzy logic in the realm of consumer products employ the fuzzy logic gambit. Basically, the fuzzy logic gambit exploits a tolerance for imprecision to achieve reduction in cost.
THE CONCEPT OF COINTENSIVE PRECISIATION

- m-precisiation of a concept or proposition, p, is cointensive if p* is cointensive with p.

Example: bear market

We classify a bear market as a 30 percent decline after 50 days, or a 13 percent decline after 145 days. (Robert Shuster)

This definition is clearly not cointensive
EXAMPLE: IMPACT FACTOR

- $A = \text{citations in 1992 to articles published in 1987-91}$

- $B = \text{articles published in 1987-91}$

- $C = \frac{A}{B} = \text{five-year impact factor}$

- Is the definition of impact factor cointensive?
Basic question

- Given a proposition, \( p \), how can \( p \) be cointesively \( mm\)-precisiated?

Key idea

- In generalized-constraint-based semantics, \( mm\)-precisiation is carried out through the use of the concept of a generalized constraint.

- What is a generalized constraint?
THE CONCEPT OF A GENERALIZED CONSTRAINT

A BRIEF INTRODUCTION
PREAMBLE

- The concept of a generalized constraint is the centerpiece of generalized-constraint-based semantics.

- In scientific theories, representation of constraints is generally oversimplified. Oversimplification of constraints is a necessity because existing constraint definition languages have a very limited expressive power.
The concept of a generalized constraint is intended to provide a basis for construction of a maximally expressive meaning precisiation language for natural languages.
GENERALIZED CONSTRAINT (Zadeh 1986)

- Bivalent constraint (hard, inelastic, categorical:)
  \[ X \in C \]
  constraining bivalent relation

- Generalized constraint on \( X \): \( GC(X) \)

\[
\begin{align*}
GC(X): X \text{ isr } R \\
\end{align*}
\]

constraining non-bivalent (fuzzy) relation

index of modality (defines semantics)

constrained variable

- \( r: \in \{ =, \leq, \geq, \subset, \ldots \} \) bivalent

primary

- open \( GC(X): X \text{ is free } (GC(X) \text{ is a predicate}) \)

- closed \( GC(X): X \text{ is instantiated } (GC(X) \text{ is a proposition}) \)
GENERALIZED CONSTRAINT—MODALITY $r$

$X isr R$

$r: = \text{equality constraint: } X = R \text{ is abbreviation of } X \text{ is } = R$

$r: \leq \text{inequality constraint: } X \leq R$

$r: \subset \text{subsethood constraint: } X \subset R$

$r: \text{blank possibilistic constraint; } X \text{ is } R; R \text{ is the possibility distribution of } X$

$r: v \text{ veristic constraint; } X \text{ isv } R; R \text{ is the verity distribution of } X$

$r: p \text{ probabilistic constraint; } X \text{ isp } R; R \text{ is the probability distribution of } X$

Standard constraints: bivalent possibilistic, bivalent veristic and probabilistic
CONTINUED

\( r: rs \) random set constraint; \( X \) isrs \( R \); \( R \) is the set-valued probability distribution of \( X \)

\( r: fg \) fuzzy graph constraint; \( X \) isfg \( R \); \( X \) is a function and \( R \) is its fuzzy graph

\( r: u \) usuality constraint; \( X \) isu \( R \) means usually (\( X \) is \( R \))

\( r: g \) group constraint; \( X \) isg \( R \) means that \( R \) constrains the attribute-values of the group
PRIMARY GENERALIZED CONSTRAINTS

- Possibilistic: X is R
- Probabilistic: X isp R
- Veristic: X isv R

Primary constraints are formalizations of three basic perceptions: (a) perception of possibility; (b) perception of likelihood; and (c) perception of truth

In this perspective, probability may be viewed as an attribute of perception of likelihood
STANDARD CONSTRAINTS

- Bivalent possibilistic: $X \in C$ (crisp set)
- Bivalent veristic: $\text{Ver}(p)$ is true or false
- Probabilistic: $X \text{isp} R$
- Standard constraints are instances of generalized constraints which underlie methods based on bivalent logic and probability theory
GENERALIZED CONSTRAINT LANGUAGE (GCL)

- GCL is an abstract language
- GCL is generated by combination, qualification, propagation and counterpropagation of generalized constraints
- examples of elements of GCL
  - X/Age(Monika) is R/young (annotated element)
  - (X isp R) and (X,Y) is S)
  - (X isr R) is unlikely) and (X iss S) is likely
  - If X is A then Y is B
- the language of fuzzy if-then rules is a sublanguage of GCL

- deduction= generalized constraint propagation and counterpropagation
CLARIFICATION

LANGUAGE VS. LANGUAGE SYSTEM

- **Language** = (description system)
- **Description system** = (syntax, semantics)
- **Language system** = (description system, computation/deduction system)
- **GCL is a language system**
THE CONCEPT OF GENERALIZED CONSTRAINT AS A BASIS FOR PRECISIATION OF MEANING

- Meaning postulate

\[ p \xrightarrow{\text{mm-precisiation}} X \xrightarrow{\text{isr}} R \]

Equivalently, mm-precisiation of \( p \) may be realized through translation of \( p \) into GCL.
A proposition, p, may be viewed as an answer to a question, q.

A question can be expressed as: What is the value of X? Where X is explicit or implicit in p.

A generalized constraint may be interpreted as an answer to a question. From this it follows that the answer to q may be expressed as a generalized constraint.

\[ X \text{ isr } R \]

In general X and R are implicit in p. In this sense, the meaning of p may be expressed as a generalized constraint in which X and R are defined procedurely.

Note that X is a variable that is focused on but is not uniquely determined by X. For this reason, X is referred to as a focal variable.
EXAMPLES: POSSIBILISTIC

- Lily is young $\rightarrow$ Age (Lily) is young

- most Swedes are tall $\rightarrow$ Count (tall.Swedes/Swedes) is most
EXAMPLES: PROBABILISTIC

- $X$ is a normally distributed random variable with mean $m$ and variance $\sigma^2$ \iff $X \sim N(m, \sigma^2)$

- $X$ is a random variable taking the values $u_1, u_2, u_3$ with probabilities $p_1, p_2$ and $p_3$, respectively \iff $X \sim (p_1 u_1 + p_2 u_2 + p_3 u_3)$
EXAMPLES: VERISTIC

- Robert is half German, quarter French and quarter Italian

  \[ \text{Ethnicity (Robert) isv (0.5|German + 0.25|French + 0.25|Italian)} \]

- Robert resided in London from 1985 to 1990

  \[ \text{Reside (Robert, London) isv [1985, 1990]} \]
THE CONCEPT OF A PROTOFORM

- Protoform of p, Pro(p): abstracted summary of p

- most Swedes are tall
- most balls are large
- QA’s are B’s
- Count(B/A) is Q
PROTOFORMAL DEDUCTION

question/proposition

protoformal transformation

Pro(q/Pro(p))

deduction

ans(q/p)

linguistic transformation

ans(Pro(q/Pro(p)))
**EXAMPLE**

- **p:** most Swedes are tall
- **q:** how many are not tall?

\[ \text{Count(tall.Swedes/Swedes) is most Count(not.tall.Swedes)/tall.Swedes is } R \]

- **q** is (1-most)
- **R** is (1-most)
Representation of the meaning of propositions as generalized constraints open the door to computation with information described in natural language, or NL-Computation for short.

Simple examples:

- Most Swedes are tall. What is the average height of Swedes?
- A box contains about 20 balls of various sizes. Most are small. There are many more small balls than large balls. What is the number of balls which are neither small nor large?
- Usually most UA flights leave on time. What is the probability that my flight will be delayed?
Representing the meaning of a proposition as a generalized constraint reduces the problem of computation with information described in natural language to the problem of computation with generalized constraints. In large measure, computation with generalized constraints involves the use of rules which govern propagation and counterpropagation of generalized constraints. Among such rules, the principal rule is the deduction principle (Zadeh 1965, 1975).
EXTENSION PRINCIPLE (POSSIBILISTIC)

- $X$ is a variable which takes values in $U$, and $f$ is a function from $U$ to $V$. The point of departure is a possibilistic constraint on $f(X)$ expressed as $f(X)$ is $A$ where $A$ is a fuzzy relation in $V$ which is defined by its membership function $\mu_A(v), \forall v \in V$.

- $g$ is a function from $U$ to $W$. The possibilistic constraint on $f(X)$ induces a possibilistic constraint on $g(X)$ which may be expressed as $g(X)$ is $B$ where $B$ is a fuzzy relation. The question is: What is $B$?
subject to

\[ \mu_B(w) = \sup_u \mu_A(f(u)) \]

\[ w = g(u) \]

\[ f(X) \text{ is } A \]

\[ g(X) \text{ is } B \]

\( \mu_A \) and \( \mu_B \) are the membership functions of \( A \) and \( B \), respectively.
STRUCTURE OF THE EXTENSION PRINCIPLE

\[ f^{-1}(A) \]

\[ u \]

\[ \mu_A(f(u)) \]

\[ f^{-1}(A) \]

\[ f(u) \]

\[ g(f^{-1}(A)) \]

\[ A \]

\[ B \]

\[ W \]

\[ f^{-1} \]

\[ f \]

\[ g \]

counterpropagation

propagation
PRECISSION AND COMPUTATION/DEDUCTION—EXAMPLE

- \( p: \) most Swedes are tall
  \( p^*: \) \( \Sigma \) Count(tall.Swedes/Swedes) is most

- \( q: \) How many are short?

Further precisiation

- \( X(h): \) height density function (not known)
  \( X(h)du: \) fraction of Swedes whose height is in \([h, h+du]\), \( a \leq h \leq b\)

\[ \int_a^b X(h)du = 1 \]
CONTINUED

- fraction of tall Swedes: \( \int_a^b X(h) \mu_{\text{tall}}(h) \, dh \)

- constraint on \( X(h) \)

\[ \int_a^b X(h) \mu_{\text{tall}}(h) \, dh \text{ is most granular value} \]

\[ \pi(X) = \mu_{\text{most}}(\int_a^b X(h) \mu_{\text{tall}}(h) \, dh) \]
CONTINUED

deduction:

\[ \int_a^b X(h) \mu_{tall}(h) \, dh \text{ is most } \leftarrow \text{ given} \]

\[ \int_a^b X(h) \mu_{short}(h) \, dh \text{ is } ? \, Q \leftarrow \text{ needed} \]

solution:

\[ \mu_Q(v) = \sup X(\mu_{most}(\int_a^b X(h) \mu_{tall}(h) \, dh)) \]

subject to

\[ v = \int_a^b X(h) \mu_{short}(h) \, dh \]

\[ \int_a^b X(h) \, dh = 1 \]
The concept of a set has a position of centrality in mathematics. The concept of a fuzzy set is a fundamental generalization of the concept of a set. In large measure, the wide-ranging impact of fuzzy logic is rooted in the generalization of a set to a fuzzy set. Among the principal contributions of fuzzy logic are the following.
Continued

- FL-Generalization
- High power of cointensive precisiation
- Precisiation of scientific concepts
- Generalized-constraint-based semantics of natural languages
- NL-Computation/Computing with Words
- Computation with imprecise probabilities
- Fuzzy logic gambit
- Computation with imprecise probabilities
- Decision-making under second-order uncertainty
CONCLUSION

- Existing scientific theories are based on bivalent logic—a logic in which everything is black or white, with no shades of gray allowed.
- What is not recognized, to the extent that it should, is that bivalent logic is in fundamental conflict with reality.
- Fuzzy logic is not in conflict with bivalent logic—it is a generalization of bivalent logic in which everything is, or is allowed to be, a matter of degree.
- Fuzzy logic provides a foundation for the methodology of computing with words and perceptions.


From computing with numbers to computing with words --from manipulation of measurements to manipulation of perceptions, IEEE Transactions on Circuits and Systems 45, 105-119, 1999.


Factual Information About the Impact of Fuzzy Logic

PATENTS

1. Number of fuzzy-logic-related patents applied for in Japan: 17,740
2. Number of fuzzy-logic-related patents issued in Japan: 4,801
3. Number of fuzzy-logic-related patents issued in the US: around 1,700

Number of papers in INSPEC and MathSciNet which have "fuzzy" in title:

<table>
<thead>
<tr>
<th>Period</th>
<th>INSPEC - &quot;fuzzy&quot; in title</th>
<th>MathSciNet - &quot;fuzzy&quot; in title</th>
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<tr>
<td>1970-1979</td>
<td>569</td>
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<td>1980-1989</td>
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<td>1990-1999</td>
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<td>2000-present</td>
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<td><strong>Total</strong></td>
<td><strong>51,096</strong></td>
<td><strong>14,612</strong></td>
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JOURNALS ("fuzzy" in title)
1. Fuzzy in title
2. Fuzzy Sets and Systems
3. IEEE Transactions on Fuzzy Systems
4. Fuzzy Optimization and Decision Making
5. Journal of Intelligent & Fuzzy Systems
6. Fuzzy Economic Review
10. International Review of Fuzzy Mathematics
11. Fuzzy Systems and Soft Computing