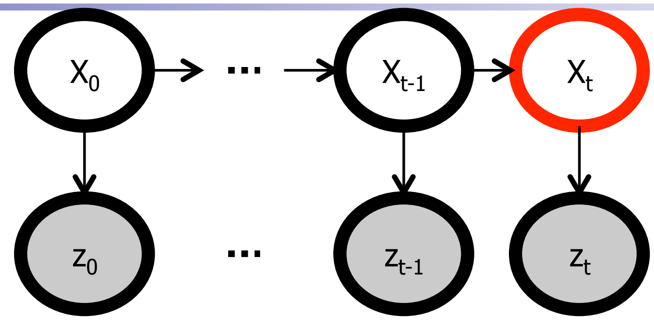
Maximum A Posteriori (MAP) Estimation

Pieter Abbeel
UC Berkeley EECS

Overview

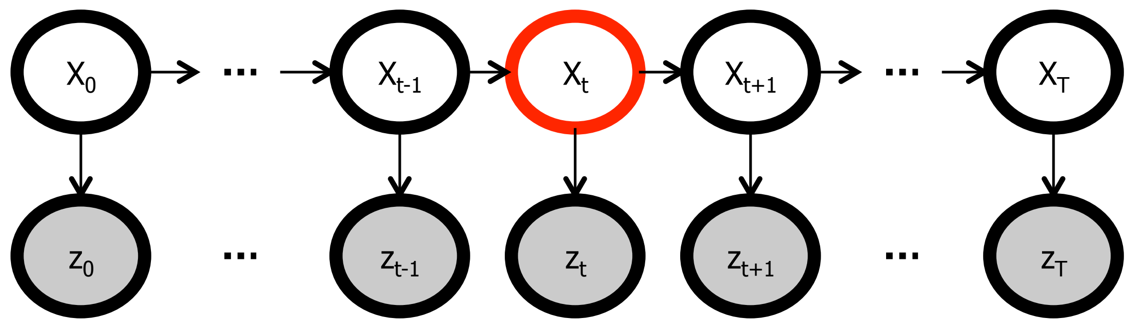
- **Filtering:**

$P(x_t | z_{0:t})$



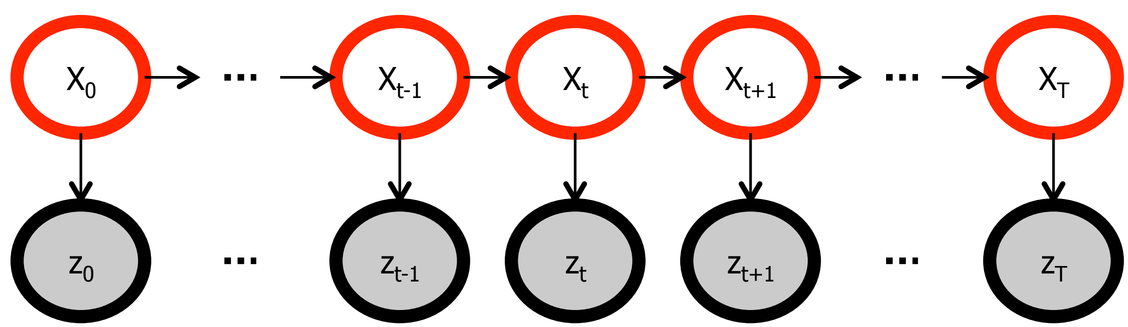
- **Smoothing:**

$P(x_t | z_{0:T})$



- **MAP:**

$\max_{x_{0:T}} P(x_{0:T} | z_{0:T})$



MAP

$$\max_{x_0,x_1,x_2,x_3} P(x_0,x_1,x_2,x_3|z_0,z_1,z_2,z_3)$$
$$\propto \max_{x_0,x_1,x_2,x_3} P(x_0,x_1,x_2,x_3,z_0,z_1,z_2,z_3)$$
$$= \max_{x_0,x_1,x_2,x_3} P(z_3|x_3)P(x_3|x_2)P(z_2|x_2)P(x_2|x_1)P(z_1|x_1)P(x_1|x_0)P(z_0|x_0)P(x_0)$$
$$= \max_{x_3}\left(P(z_3|x_3)\max_{x_2}\left(P(x_3|x_2)P(z_2|x_2)\max_{x_1}\left(P(x_2|x_1)P(z_1|x_1)\max_{x_0}\left(P(x_1|x_0)P(z_0|x_0)P(x_0)\right)\right)\right)\right)$$

$m_0(x_0)$

$m_1(x_1)$

$m_2(x_2)$

$m_3(x_3)$

Naively solving by enumerating all possible combinations of x_0,...,x_T is exponential in T !

- Generally: $$m_t(x_t) = \max_{x_{0:t-1}} P(x_{0:t}, z_{0:t})$$
$$= \max_{x_{0:t-1}} P(x_t|x_{t-1})P(z_t|x_t)P(x_{0:t-1}, z_{0:t-1})$$
$$= P(z_t|x_t)\max_{x_{t-1}} P(x_t|x_{t-1}) \max_{x_{0:t-2}} P(x_{0:t-1}, z_{0:t-1})$$
$$= P(z_t|x_t)\max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}(x_{t-1})$$

MAP --- Complete Algorithm

1. Init: $m_0(x_0) = P(z_0|x_0)P(x_0)$
2. For all $t = 1, 2, \ldots, T-1$
   - For all x_t: $m_t(x_t) = P(z_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}(x_{t-1})$
   - For all x_t: Store argmax in $\text{pointer}_{t \to t-1}(x_t)$
3. $\text{maximum} = \max_{x_T} m_T(x_T)$
4. $x_T^* = \arg\max_{x_T} m_T(x_T)$
5. For all $t = T, T-1, \ldots, 1$
   - $x_{t-1}^* = \text{pointer}_{t \to t-1}(x_t^*)$

- $O(T\, n^2)$

Kalman Filter (aka Linear Gaussian) setting

- Summations → integrals
- But: can't enumerate over all instantiations
- However, we can still find solution efficiently:
  - the joint conditional $P(x_{0:T} \mid z_{0:T})$ is a multivariate Gaussian
  - for a multivariate Gaussian the most likely instantiation equals the mean

  → we just need to find the mean of $P(x_{0:T} \mid z_{0:T})$
    - the marginal conditionals $P(x_t \mid z_{0:T})$ are Gaussians with mean equal to the mean of x_t under the joint conditional, so it suffices to find all marginal conditionals
    - We already know how to do so: marginal conditionals can be computed by running the Kalman smoother.
- Alternatively: solve convex optimization problem