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This expression 'counts' edges in cut 'x' plus scales by volume.
Yields $h(S)$.

Hypercube

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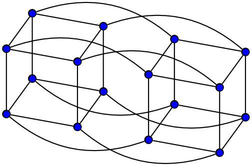

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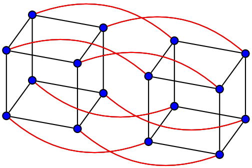

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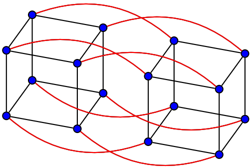

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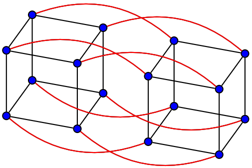

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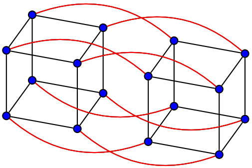

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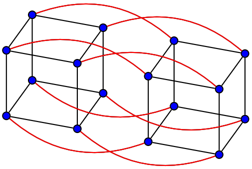

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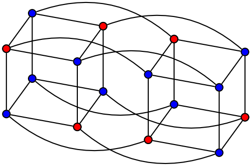

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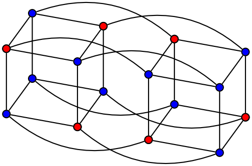

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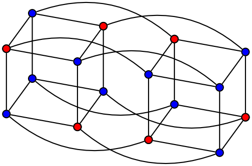

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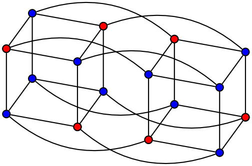

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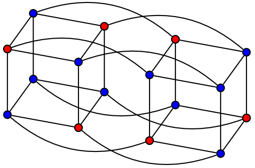

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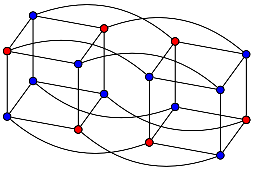

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Worse by a factor of $\sqrt{d}$

Eigenvalues of hypercube.

Anyone see any symmetry?

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Delete edges in two dimensions.

Four subcubes: bipartite. Color ± 1

Eigenvalue:

Eigenvalues of hypercube.

Anyone see any symmetry?

Coordinate cuts. +1 on one side, -1 on other.

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Back to Cheeger.

Coordinate Cuts:

Back to Cheeger.

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Eigenvalue $1-2/d$.

Back to Cheeger.

Coordinate Cuts:
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$\frac{\mu}{2}$

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Coordinate Cuts:
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For hypercube: $h(G) = \frac{1}{d}$

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Left hand side is tight.

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Cut along line.

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Tight example for Other side of Cheeger?

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Tight example for upper bound for Cheeger.

Eigenvalues of cycle?

Eigenvalues: $\cos \frac{2\pi k}{n}$.

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$(Mx)_i$

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$(Mx)_i = \cos\left(\frac{2\pi k(i+1)}{n}\right) + \cos\left(\frac{2\pi k(i-1)}{n}\right)$

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Eigenvalues:
vibration modes of system.

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Random Walk.

p - probability distribution.

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Probability distrubtion after choose a random neighbor.

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$M^t x = a_1 \lambda_1^t v_1 + a_2 \lambda_2 v_2 + \cdots$

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$\Omega(n^2)$ steps to get close to uniform.

Random Walk.

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Probability distrubtion after choose a random neighbor.
Mp.

Converge to uniform distribution.

Power method: $M^t x$ goes to highest eigenvector.

$M^t x = a_1 \lambda_1^t v_1 + a_2 \lambda_2 v_2 + \cdots$

$\lambda_1 - \lambda_2$ - rate of convergence.

$\Omega(n^2)$ steps to get close to uniform.

Start at node 0, probability distribution, $[1, 0, 0, \cdots, 0]$.
Takes $\Omega(n^2)$ to get n steps away.

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Recall drunken sailor.

Eigenvalues, random walks, volume estimation, counting.

Sampling.

Sampling: Random element of subset $S \subset \{0,1\}^n$ or $\{0,\ldots,k\}^k$.

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Related Problem: Approximate $|S|$ within factor of $1+\varepsilon$.

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Random walk to do both for some interesting sets S.

Convex Bodies.

$S \subset [k]^n$ is grid points inside Convex Body.

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Ex: Numerically integrate convex function in d dimensions.

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But P could be exponentially small compared to $|[k]^n|$.
Takes a long time to even find a point in P.

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How long does this take? More later.

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But remember power method...

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But remember power method...which finds first eigenvector.

Spanning Trees.

Problem: How many?

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Another Problem: find a random one.

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Algorithm:

Spanning Trees.

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Algorithm:
Start with spanning tree.

Spanning Trees.

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Algorithm:
  Start with spanning tree.
  Repeat:

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Algorithm is random walk on BIG GRAPH (sample space graph.)

Spin systems.

Each element of S may have associated weight.

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Sample element proportional to weight.

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Spin systems.

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Example?
2 or 3 dimensional grid of particles.

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 Particle State ± 1.

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Particle State ± 1. System State $\{-1,+1\}^n$.

Energy on local interactions: $E = \sum_{(i,j)} -\sigma_i \sigma_j$.

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Metropolis Algorithm:

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Random walk in sample space graph (BIG GRAPH ALERT)
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Markov Chain on statespace of system.

Sampling structures and the BIG GRAPH

Sampling Algorithms $\equiv$ Random walk on BIG GRAPH.

Sampling structures and the BIG GRAPH

Sampling Algorithms $\equiv$ Random walk on BIG GRAPH. Small degree.

Vertices

Grid points in convex body.

Neighbors

Degree (ish)

Sampling structures and the BIG GRAPH

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Vertices — Grid points in convex body.

Neighbors — Change one dimension

Degree (ish)

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Vertices	Neighbors	Degree (ish)
Grid points in convex body.	Change one dimension	$2d$
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Spanning Trees.	Change two edges.	$\leq \|V\|^2$ neighbors per node
Spin States.	Change one spin	$O(n)$ neighbors.

Analyzing random walks on graph.

Start at vertex, go to random neighbor.

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How to analyse?

Analyzing random walks on graph.

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How to analyse?

Random Walk Matrix: M.

Analyzing random walks on graph.

Start at vertex, go to random neighbor.
  For d-regular graph: eventually uniform.
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How to analyse?

Random Walk Matrix: M.
  M - normalized adjacency matrix.

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For d-regular graph: eventually uniform.
if not bipartite. Odd /even step!

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Rapidly mixing with big ($\geq \frac{1}{p(n)}$) spectral gap.

Rapid mixing, volume, and surface area..

Recall volume of convex body.

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  Grid graph on grid points inside convex body.

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Lower bound expansion

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Lower bound expansion $\rightarrow$ lower bounds on spectral gap μ

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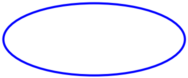

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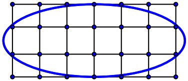

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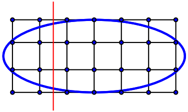

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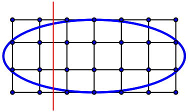

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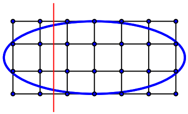

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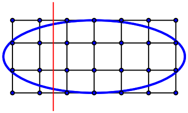

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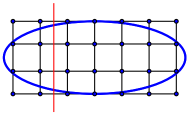

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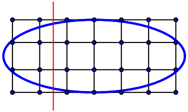

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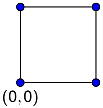

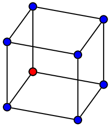

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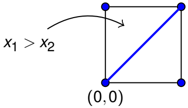

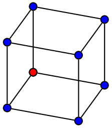

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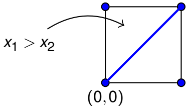

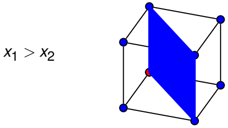

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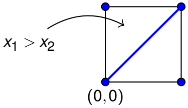

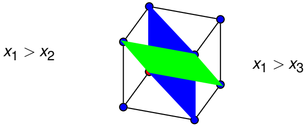

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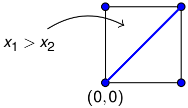

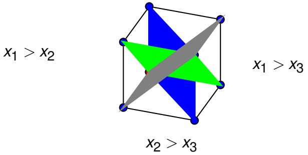

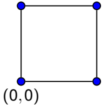
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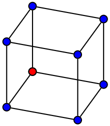

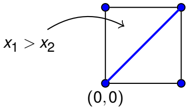
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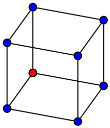

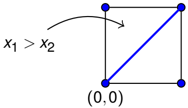
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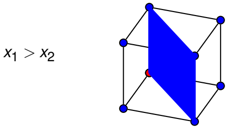
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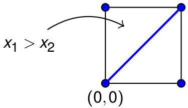
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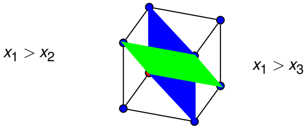
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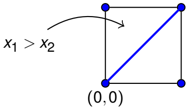
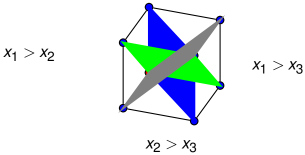
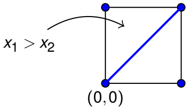
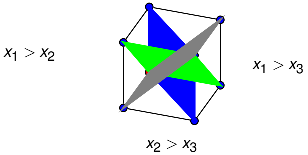




Each order takes $\frac{1}{n!}$ volume.





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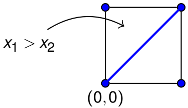
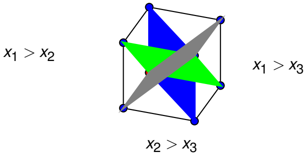
Number of orders $\equiv$ volume of intersection of partial order relations.





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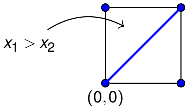

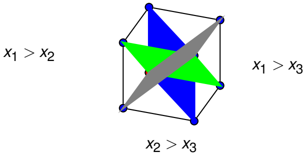

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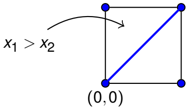

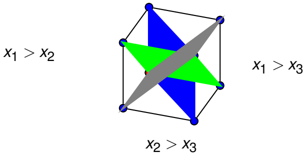

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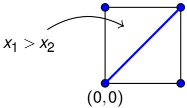

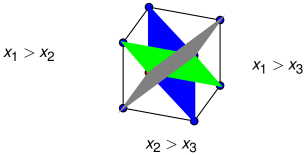

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Number of orders $\equiv$ volume of intersection of partial order relations.

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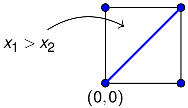

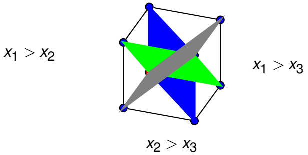

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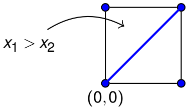

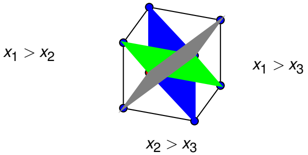

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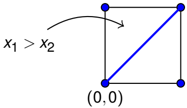

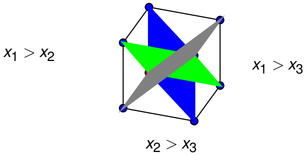

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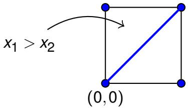

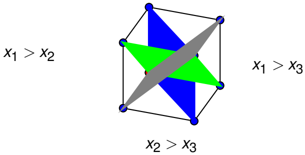

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Do the polynomial dance!!!

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Partial Order Application.