1. Write the 4×4 matrix of the unitary operation on two qubits resulting from performing a Hadamard transform on the first qubit and a phase flip on the second qubit.

2. Consider the unitary operation U resulting from applying the Hadamard gate to each of n qubits. Describe U by giving a formula for its $(x, y)^{th}$ entry.

3. Consider a CNOT gate whose second input is $|0\rangle - |1\rangle$. Describe the action of the CNOT gate on the first qubit.

   Now show that if the CNOT gate is applied in the Hadamard basis - i.e. apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubit swapped.

4. Show that if U and V are unitary, then so is $U \otimes V$.

5. You are given one of two quantum states of a single qubit: either $|\phi\rangle = |0\rangle$ or $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$. What measurement best distinguishes between these two states? If the state you are presented is either $|\phi\rangle$ or $|\psi\rangle$ with 50% probability each, what is the probability that your measurement correctly identifies the state? Can you generalize your result to distinguish between two arbitrary quantum states $|\phi\rangle$ and $|\psi\rangle$ on two qubits?

6. Alice and Bob share an arbitrarily long common string S. Alice is given as input a random bit x_A and Bob a random bit x_B. Without communicating with each other, Alice and Bob wish to output bits a and b respectively such that $x_A \wedge x_B = a \oplus b$. Prove that any protocol that Alice and Bob follow has success probability at most 3/4.

7. Prove that the Bell state $|\psi^-\rangle$ is rotationally invariant: i.e. $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|vv^\perp\rangle - |v^\perp v\rangle)$.