5 Functions of Functions

We now have many of the tools we need to write computer programs. We have the primitive operations for arithmetic computation, the primitive operations to manipulate words and sentences, and a way to choose between alternative computations. One thing that we still lack is a way to deal systematically with data *aggregates*—collections of data. We want to be able to say "carry out this computation for each member of that aggregate." Processing large amounts of data uniformly is one of the abilities that distinguish computers from mere pocket calculators.

The Problem: `Initials`

To make this concrete, we'll look at a very simple example. I'd like to write a procedure that can figure out a person's initials, like this:

```
? show initials [George Harrison]
[G H]
```

One obvious approach is to find the initials of the first name and the last name:

```
to initials :name
output sentence (first first :name) (first last :name)
end
```

The trouble is that this approach doesn't work for people with middle names. We'd like our `initials` procedure to be able to handle any length name. But it doesn't:

```
? show initials [John Alec Entwistle]
[J E]
? show initials [Peter Blair Denis Bernard Noone]
[P N]
```

What we want is this:

```
? show initials.in.our.dreams [John Alec Entwistle]
[J A E]
? show initials.in.our.dreams [Peter Blair Denis Bernard Noone]
[P B D B N]
```

If we knew that the input would have exactly five names, we could extract the first letter of each of them explicitly. But you never know when some smart alec will ask you to

```
show initials [Princess Angelina Contessa Louisa Francesca ~
               Banana Fana Bo Besca the Third]
```

One Solution: Numeric Iteration

If you've programmed before in other languages, then one solution will immediately occur to you. You create a variable `n` whose value is the number of words in the input, then you have a variable `i` that takes on all possible values from 1 to `n`, and you select the `i`th word from the input and pull out its first letter. Most languages have a special notation for this sort of computation:

```
for i = 1 to n : ... : next i                 (BASIC)
for 1 := 1 to n do begin ... end              (Pascal)
for (i=1; i<=n; i++) { ... }                  (C)
```

All of these have the same meaning: Carry out some instructions (the part shown as `...` above) repeatedly, first with the variable named `i` having the value `1`, then with `i` equal to `2`, and so on, up to `i` equal to `n`. This technique is called *numeric iteration.* "Iteration" means repetition, and it's "numeric" iteration because the repetition is controlled by a variable that takes on a sequence of numeric values.

We can do the same thing in Logo, although, as we'll soon learn, it's not the usual approach that Logo programmers take to this problem.

```
to initials :name
local "result
make "result []
for [i 1 [count :name]] ~
    [make "result sentence :result first (item :i :name)]
output :result
end
```

(The reason I declare `result` as local, but not `i`, is that Logo's `for` automatically makes its index variable local to the `for` itself. There is no variable `i` outside of the `for` instruction.)

The command `for` takes two inputs. The second input is an instruction list that will be carried out repeatedly. The first input controls the repetition; it is a list of either three or four members: a variable name, a starting value, a limit value, and an optional increment. (The variable named by the first member of the list is called the *index variable*. For example:

```
? for [number 4 7] [print :number]
4
5
6
7
? for [value 4 11 3] [print :value]
4
7
10
```

In the first example, `number` takes on all integer values between 4 and 7. In the second, `value`'s starting value is 4, and on each repetition its new value is 3 more than last time. `Value` never actually has its limiting value of 11; the next value after 10 would have been 13, but that's bigger than the limit.

`For` can count downward instead of upward:

```
? for [i 7 5] [print :i]
7
6
5
? for [n 15 2 -6] [print :n]
15
9
3
? for [x 15 2 6] [print :x]
?
```

The last example has no effect. Why? The increment of 6 implies that this invocation of `for` should count upward, which means that the `for` continues until the value of `x` is greater than the limit, 2. But the starting value, 15, is *already* greater than 2.

If no increment is given in the first input to `for`, then `for` will use either `1` or `-1` as the increment, whichever is compatible with the starting and limit values.

Although I've been using constant numbers as the starting value, limit value, and increment in these examples, `for` can handle any Logo expression, represented as a list, for each of these:

```
to spread :ends
for [digit [first :ends] [last :ends]] [type :digit]
print []
end

? spread 19
123456789
? spread 83
876543
```

More formally, the effect of `for` is as follows. First it creates the local index variable and assigns it the starting value. Then `for` carries out three steps repeatedly: testing, action, and incrementing. The testing step is to compare the current value of the index variable with the limit value. If the index variable has passed the limit, then the `for` is finished. ("Passed" means that the index variable is greater than the limit, if the increment is positive, or that the index variable is less than the limit, if the increment is negative.) The action step is to evaluate the instructions in the second input to `for`. The incrementing step is to assign a new value to the index variable by adding the increment to the old value. Then comes another round of testing, action, and incrementing.

So, for example, if we give Logo the instruction

```
show initials [Raymond Douglas Davies]
```

then the `for` instruction within `initials` is equivalent to this sequence of instructions:

```
local "i                                ; initialize index variable
make "i 1

if (:i > 3) [stop]                      ; testing
make "result (se :result first "Raymond)  ; action  (result is [R])
make "i :i+1                            ; incrementing  (i is 2)

if (:i > 3) [stop]                      ; testing
make "result (se :result first "Douglas)  ; action  (result is [R D])
make "i :i+1                            ; incrementing  (i is 3)
```

```
if (:i > 3) [stop]                       ; testing
make "result (se :result first "Davies)   ; action  (result is [R D D])
make "i :i+1                             ; incrementing  (i is 4)

if (:i > 3) [stop]                       ; testing
```

except that the `stop` instruction in the testing step stops only the `for` instruction, not the `initials` procedure.

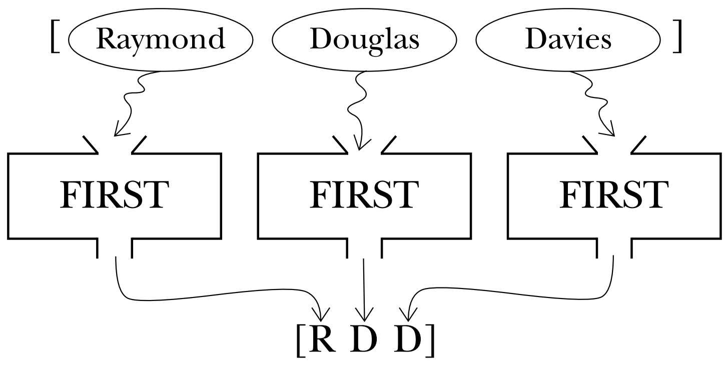
Critique of Numeric Iteration

Computers were originally built to deal with numbers. Numeric iteration matches closely the behind-the-scenes sequence of steps by which computers actually work. That's why just about every programming language supports this style of programming.

Nevertheless, a `for` instruction isn't anything like the way you, a human being, would solve the `initials` problem without a computer. First of all, you wouldn't begin by counting the number of words in the name; you really don't have to know that. You'd just say, for example, "First of Raymond is R; first of Douglas is D; first of Davies is D." When you ran out of names, you'd stop.

The manipulation of the `result` variable to collect the results also seems unnatural. You wouldn't think, "I'm going to start with an empty result; then, whatever value `result` has, I'll throw in an R; then, whatever value `result` now has, I'll throw in a D" and so on.

In fact, if you had to explain to someone else how to solve this problem, you probably wouldn't talk about a sequence of steps at all. Rather, you'd draw a picture like this one:



To explain the picture, you'd say something like "Just take the `first` of each word." You wouldn't even mention the need to put the results together into a sentence; you'd take that for granted.

In Logo we can write an `initials` procedure using the same way of thinking that you'd use in English:

```
to initials :name
output map "first :name
end
```

The `map` procedure means "collect the results of doing *this* for each of *those.*"

As this example illustrates, `map` is easy to use. But it's a little hard to talk about, because it's a function of a function. So first we'll take a detour to talk more precisely about functions in general.

What's a Function?

A *function* is a rule for turning one value (called the *argument*) into another. If you've studied algebra you'll remember numeric function rules such as

$$f(x) = 3x - 6$$

but not all functions are numeric, and not all rules need be expressed as algebraic formulas. For example, here is the Instrument function, which takes a Beatle as its argument and returns his instrument:

argument	**result**
John	rhythm guitar
Paul	bass guitar
George	lead guitar
Ringo	drums

This particular function has only four possible arguments. Other functions, like $f(x)$ above, may have infinitely many possible arguments. The set of possible arguments is called the *domain* of the function. Similarly, the set of possible result values is called the *range* of the function.*

* It's a little awkward to talk about the domain of a function that takes two arguments. That is, it's easy to say that the domain of the function represented by the `first` operation is words or lists, but how do we describe `item`? We could loosely say "its domain is numbers and words or lists," but that sounds as if either argument could be any of those. The most precise way to say it is this: "The

Functions can be represented in many ways. (We've seen two in this section: formulas and tables.) One way to represent a function is with a Logo operation. Here are Logo representations of the two functions we've discussed:

```
to f :x
output 3*:x - 6
end

to instrument :beatle
if :beatle = "John [output [rhythm guitar]]
if :beatle = "Paul [output [bass guitar]]
if :beatle = "George [output [lead guitar]]
if :beatle = "Ringo [output [drums]]
end
```

(What if we give `instrument` an input that's not in the domain of the function? In that case, it won't output any value, and a Logo error message will result. Some people would argue that the procedure should provide its own, more specific error message.)

I've been careful to say that the Logo operation *represents* the function, not that it *is* the function. In particular, two Logo procedures can compute the same function—the same relationship between input and output values—by different methods. For example, consider these Logo operations:

```
to f :x                        to g :x
output 3*:x - 6                output 3 * (:x-2)
end                            end
```

The Logo operations `f` and `g` carry out two different computations, but they represent the same function. For example, to compute `f 10` we say $3 \times 10 = 30$, $30 - 6 = 24$; to compute `g 10` we say $10 - 2 = 8$, $3 \times 8 = 24$. Different computations, but the same answer. Functional programming means, in part, focusing our attention on the inputs and outputs of programs rather than on the sequence of computational steps.

Just as a Logo operation represents a function, the procedure's inputs similarly *represent* the arguments to the corresponding function. For example, that instrument function I presented earlier has Beatles (that is to say, people) as its domain and has

domain of `item` is pairs of values, in which the first member of the pair is a positive integer and the second member is a word or list of length greater than or equal to the first member of the pair." But for ordinary purposes we just rephrase the sentence to avoid the word "domain" altogether: "`Item` takes two inputs; the first must be a positive integer and the second must be a word or list..."

musical instruments as its range. But Logo doesn't have people or instruments as data types, and so the procedure `instrument` takes as its input *the name of* a Beatle (that is, a word) and returns as its output *the name of* an instrument (a sentence). Instrument is a function from Beatles to instruments, but `instrument` is an operation from words to sentences.

We're about to see a similar situation when we explore `map`. The map function—that is, the function that `map` represents—is a *function of functions.* One of the arguments to the map function is itself a function. The corresponding input to Logo's `map` procedure should be a procedure. But it turns out that Logo doesn't quite allow a procedure to be an input to another procedure; instead, we must use the *name* of the procedure as the input, just as we use the name of a Beatle as the input to `instrument`.

I know this sounds like lawyer talk, and we haven't written any programs for a while. But here's why this is important: In order to understand the *purpose* of `map`, you have to think about the map function, whose domain is functions (and other stuff, as we'll see in a moment). But in order to understand the *notation* that you use with `map` in Logo, you have to think in terms of the Logo operation, whose input is words (names of procedures). You have to be clear about this representation business in order to be able to shift mentally between these viewpoints.

Functions of Functions: `Map`

`Map` takes two inputs. The first is a word, which must be the name of a one-input Logo operation. The second can be any datum. The output from `map` is either a word or a list, whichever is the type of the second input. The members of the output are the results of applying the named operation to the members of the second input.

```
? show map "first [Rod Argent]
[R A]
```

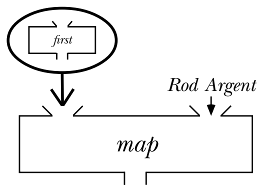
In this example, the output is a list of two members, just as the second input is a list of two members. Each member of the output is the result of applying `first` to one of the members of `map`'s second input.

Many people, when they first meet `map`, are confused by the quoting of its first input. After all, I made a fuss back in Chapter 2 about the difference between these two examples:

```
? print Hello
I don't know how  to Hello
? print "Hello
Hello
```

You learned that a quoted word means the word itself, while an unquoted word asks Logo to invoke a procedure. But now, when I want to use the `first` procedure as input to `map`, I'm quoting its name. Why?

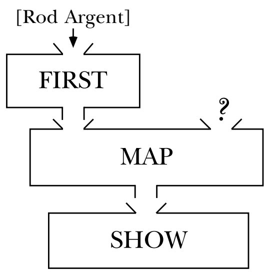
All that effort about the domains of functions should help you understand the notation used here. Start by ignoring the Logo notation and think about the domain of the map function. We want the map function to have *another function,* the function "first" in this case, as one of its arguments:



It's tempting to say that in Logo, a function is represented by a procedure, so `map` represents map, and `first` represents first. If this were algebra notation, I'd say *map*(*first*, *Rod Argent*), so in Logo I'll say

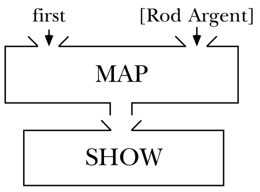
```
show map first [Rod Argent]           ;; wrong!
```

But when a Logo instruction has two unquoted procedure names in a row, that doesn't mean that the second function is used as argument to the first! Instead, it means that *the output from invoking* the second function is used as the argument to the first. In this case, we'd be *composing* `map` and `first`:



As the plumbing diagram shows, the list that we intended as the second input to `map` actually ends up as the input to `first`, and Logo will complain because `map` isn't given enough inputs.

Instead, as I said earlier, we must use *the name of* the `first` procedure to represent it. That gives this diagram:



Here's another simple example. Logo has a primitive operation `uppercase` that takes a word as input, and outputs the same word but in all capital letters:

```
? print uppercase "young
YOUNG
```

What if we want to translate an entire sentence to capital letters? The `uppercase` primitive doesn't accept a sentence as its input:

```
? show uppercase [neil young]
uppercase doesn't like [neil young] as input.
```

But we can use `map` to translate each word separately and combine the results:

```
? show map "uppercase [neil young]
[NEIL YOUNG]
```

Ordinarily `map` works with one-argument functions. But we can give `map` extra arguments (by enclosing the invocation of `map` in parentheses, as usual) so that it can work with functions of more than one argument.

```
? show (map "item [2 1 2 3] [john paul george ringo])
[o p e n]
? show (map "sum [1 2 3] [40 50 60] [700 800 900])
[741 852 963]
```

Each input after the first provides values for one input to the mapped function. For example, `[2 1 2 3]` provides four values for the first input to `item`. The input lists must all have the same length (two lists of length four in the `item` example, three lists of length three in the `sum` example).

In the examples so far, the input data have been lists. Here's an example in which we use `map` with words. Let's say we're writing a program to play Hangman, the word game in which one player guesses letters in a secret word chosen by the other player. At first the guesser sees only a row of dashes indicating the number of letters in the word; for each guess, more letters are revealed. We aren't going to write the entire program yet, but we're ready to write the operation that takes the secret word, and a list of the letters that have been guessed so far, and outputs a row of letters and dashes as appropriate.

```
to hangword :secret :guessed
output map "hangletter :secret
end

to hangletter :letter
output ifelse memberp :letter :guessed [:letter] ["_]
end

? print hangword "potsticker [e t a o i n]
_ot_ti__er
? print hangword "gelato [e t a o i n]
_e_ato
```

Notice that `hangletter` depends on Logo's dynamic scope to have access to `hangword`'s local variable named `guessed`.

☞ Write an operation `exaggerate` that takes a sentence as input and outputs an exaggerated version:

```
? print exaggerate [I ate 3 potstickers]
I ate 6 potstickers
? print exaggerate [The chow fun is good here]
The chow fun is great here
```

It should double all the numbers in the sentence, and replace "good" with "great," "bad" with "terrible," and so on.

A function whose domain or range includes functions is called a *higher order function.* The function represented by `map` is a higher order function. (We may speak loosely and say that `map` is a higher order function, as long as you remember that Logo procedures aren't really functions!) It's tempting to say that the `map` procedure itself is a "higher order procedure," but in Logo that isn't true. Procedures aren't data in Logo; the only data types are words and lists. That's why the input to `map` is a word, the name of a procedure, and not the procedure itself. Some languages do treat procedures themselves as data. In particular, the language Scheme is a close relative of Logo that can handle procedures as data. If this way of thinking appeals to you, consider learning Scheme next!

Higher Order Selection: `Filter`

The purpose of `map` is to *transform* each member of an aggregate (a list or a word) by applying some function to it. Another higher order function, `filter`, is used to *select* some members of an aggregate, but not others, based on a criterion expressed as a predicate function. For example:

```
? show filter "numberp [76 trombones, 4 calling birds, and 8 days]
[76 4 8]

to vowelp :letter
output memberp :letter "aeiou
end

? show filter "vowelp "spaghetti
aei

to beatlep :person
output memberp :person [John Paul George Ringo]
end

? show filter "beatlep [Bob George Jeff Roy Tom]
[George]
```

What happens if we use the `initials` procedure that we wrote with people's names in mind for other kinds of names, such as organizations or book titles? Some of them work well:

```
? show initials [Computer Science Logo Style]
[C S L S]
? show initials [American Civil Liberties Union]
[A C L U]
```

but others don't give quite the results we'd like:

```
? show initials [Association for Computing Machinery]
[A f C M]
? show initials [People's Republic of China]
[P R o C]
```

We'd like to eliminate words like "for" and "of" before taking the first letters of the remaining words. This is a job for `filter`:

```
to importantp :word
output not memberp :word [the an a of for by with in to and or]
end

to initials :name
output map "first (filter "importantp :name)
end

? show initials [Association for Computing Machinery]
[A C M]
? show initials [People's Republic of China]
[P R C]
```

Many to One: `Reduce`

Of course, what we'd *really* like is to have those initials in the form of a single word: ACLU, CSLS, ACM, and so on. For this purpose we need yet another higher order function, one that invokes a combining function to join the members of an aggregate.

```
? show reduce "word [C S L S]
CSLS
? show reduce "sum [3 4 5 6]
18
? show reduce "sentence "UNICEF
[U N I C E F]
```

`Reduce` takes two inputs. The first must be the name of a two-input operation; the second can be any *nonempty* word or list.

```
to acronym :name
output reduce "word initials :name
end
```

In practice, the first input to `reduce` won't be any old operation; it'll be a *constructor.* It'll be something that doesn't care about the grouping of operands; for example, `sum` is a good choice but `difference` is problematic because we don't know whether

```
reduce "difference [5 6 7]
```

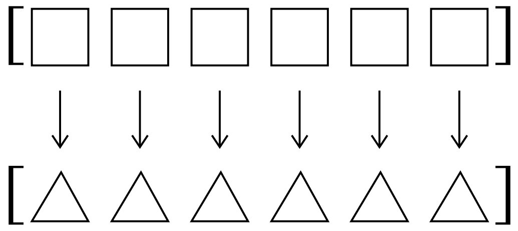
means 5 – (6 – 7) or (5 – 6) – 7, and the grouping affects the answer. Almost all the time, the constructor will be `word`, `sentence`, `sum`, or `product`. But here's an example of another one:

```
to bigger :a :b
output ifelse :a > :b [:a] [:b]
end

to biggest :nums
output reduce "bigger :nums
end

? show biggest [5 7 781 42 8]
781
```

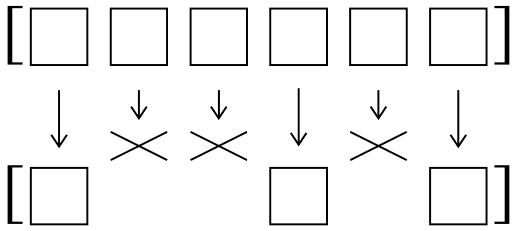
Choosing the Right Tool

So far you've seen three higher order functions: `map`, `filter`, and `reduce`. How do you decide which one to use for a particular problem?

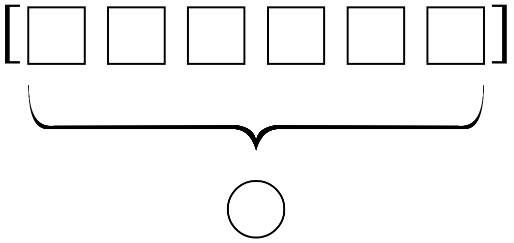
`Map` transforms each member of a word or list individually. The result contains as many members as the input.



Filter selects certain members of a word or list and discards the others. The members of the result are members of the input, without transformation, but the result may be smaller than the original.



Reduce transforms the entire word or list into a single result by combining all of the members in some way.



Anonymous Functions

In several of the examples in this chapter, I've had to write "helper" procedures such as hangletter, importantp, and bigger that will never be used independently, but are needed only to provide the function argument to a higher order function. It would be simpler if we could avoid writing these as separate procedures.

Does that sound confusing? This is one of those ideas for which an example is worth 1000 words:

```
to hangword :secret :guessed
output map [ifelse memberp ? :guessed [?] ["_]] :secret
end
```

Until now, the first input to `map` has always been a word, used to represent the function with that word as its name. In this example we see how a nameless function can be represented: as a list containing a Logo expression, but with question marks where the function's argument belongs. Such a list is called a *template.*

```
? show filter [memberp ? [John Paul George Ringo]] ~
              [Bob George Jeff Roy Tom]
[George]
```

Anonymous functions of more than one argument are a little uglier. Instead of `?` for the argument, you must use `?1` for the first, `?2` for the second, and so on.

```
to biggest :nums
output reduce [ifelse ?1 > ?2 [?1] [?2]] :nums
end
```

Notice that the templates don't say `output`, as the named procedures did. That's because procedures are made of *instructions,* whereas these are *expression* templates. When input values are "plugged in" for the question marks, the template becomes a Logo expression, which means that when evaluated it has a value. If the template said `output`, it would be saying to use that value as the output *from the procedure containing it!* (I'm just repeating the point made earlier that `output` immediately stops the procedure it's in, even if there are more instructions below it.)

Higher Order Miscellany

`Map` combines the partial results into a list, if the second argument is a list, or into a word, if it's a word. Sometimes this behavior isn't quite what you want. An alternative is `map.se` (map to sentence), which makes a sentence of the results. Here are some examples.

```
? make "numbers [zero one two three four five six seven eight nine]
? show map [item ?+1 :numbers] 5789
fiveseveneightnine
? show map.se [item ?+1 :numbers] 5789
[five seven eight nine]

? show map [sentence (word "With ?) "You] [in out]
[[Within You] [Without You]]
? show map.se [sentence (word "With ?) "You] [in out]
[Within You Without You]
```

```
? show map.se [sentence ? "Warner] [Yakko Wakko Dot]
[Yakko Warner Wakko Warner Dot Warner]
? show map [sentence ? "Warner] [Yakko Wakko Dot]
[[Yakko Warner] [Wakko Warner] [Dot Warner]]
```

As these examples show, sometimes `map` does what you want, but sometimes `map.se` does, depending on the "shape" you want your result to have. Do you want a word, a sentence, or a structured list?

Suppose we have two sets of things, and we want all the pairings of one of these with one of those. An example will make clear what's desired:

```
? show crossproduct [red blue green] [shirt pants]
[[red shirt] [blue shirt] [green shirt] [red pants] [blue pants]
 [green pants]]
```

This is a tricky example because there are two different mistakes we could make. We don't want to "flatten" the result into a sentence:

```
[red shirt blue shirt green shirt red pants blue pants green pants]
```

but we also don't want all the shirts in one list and all the pants in another:

```
[[[red shirt] [blue shirt] [green shirt]]
 [[red pants] [blue pants] [green pants]]]
```

Here's the solution:

```
to crossproduct :these :those
output map.se [prepend.each :these ?] :those
end

to prepend.each :these :that
output map [sentence ? :that] :these
end
```

☞ Notice that this solution uses both `map` and `map.se`. Try to predict what would happen if you used `map` both times, or `map.se` both times, or interchanged the two. Then try it on the computer and be sure you understand what happens and why!

By the way, this is a case in which we still need a named helper function despite the use of templates, because otherwise we'd have one template inside the other, and Logo couldn't figure out which `?` to replace with what:

```
to crossproduct :these :those
output map.se [map [sentence ? ?] :these] :those     ; (wrong!)
end
```

Just as `map.se` is a variant of `map`, `find` is a variant of `filter`, for the situations in which you only want to find *one* member that meets the criterion, rather than all the members. (Perhaps you know in advance that there will only be one, or perhaps if there are more than one, you don't care which you get.)

```
to spellout :card
output (sentence (butlast :card) "of
                 (find [equalp last :card first ?]
                       [hearts spades diamonds clubs]))
end

? print spellout "5d
5 of diamonds
? print spellout "10h
10 of hearts
```

Sometimes what you want isn't a function at all. You want to take some *action* for each member of an aggregate. The most common one is to print each member on a separate line, in situations where you've computed a long list of things. You can use `foreach` with an *instruction* template, rather than an expression template as used with the others. The template is the last argument, rather than the first, to follow the way in which the phrase "for each" is used in English: For each of these things, do that.

```
? foreach (crossproduct [[ultra chocolate] pumpkin [root beer swirl]
      ginger] [cone cup]) "print
ultra chocolate cone
pumpkin cone
root beer swirl cone
ginger cone
ultra chocolate cup
pumpkin cup
root beer swirl cup
ginger cup
```

If you look closely at the letters on your computer screen you'll see that they are made up of little dots. One simple pattern represents each letter in a rectangle of dots five wide and seven high, like this:

```
  *    *****  *****  ****   *****
 * *   *   *  *      *   *  *
*   *  *   *  *      *   *  *
*****  ****   *      *   *  ***
*   *  *   *  *      *   *  *
*   *  *   *  *      *   *  *
*   *  *****  *****  ****   *****
```

The following program allows you to spell words on the screen in big letters like these. Each letter's shape is kept as the value of a global variable with the letter as its name. (I haven't actually listed all 26 letters.) The value is a list of seven words, each of which contains five characters, some combination of spaces and asterisks.

```
to say :word
for [row 1 7] [foreach :word [sayrow :row ?] print []]
print []
end

to sayrow :row :letter
type item :row thing :letter
type "|  |
end

make "b [|*****| |*   *| |*   *| |**** | |*   *| |*   *| |*****|]
make "r [|*****| |*   *| |*   *| |*****| |* *  | |*  * | |*   *|]
make "i [|*****| |  *  | |  *  | |  *  | |  *  | |  *  | |*****|]
make "a [|  *  | | * * | |*   *| |*****| |*   *| |*   *| |*   *|]
make "n [|*   *| |**  *| |**  *| |* * *| |*  **| |*  **| |*   *|]
```

```
? **say "brian**
*****  *****  *****    *    *   *
*   *  *   *    *     * *   **  *
*   *  *   *    *    *   *  **  *
****   *****    *    *****  * * *
*   *  * *      *    *   *  *  **
*   *  *  *     *    *   *  *  **
*****  *   *  *****  *   *  *   *
```

☞ Modify the program so that `say` takes another input, a number representing the size in which you want to print the letters. If the number is 1, then the program should work as before. If the number is 2, each dot should be printed as a two-by-two square of spaces or asterisks; if the number is 3, a three-by-three square, and so on.

Repeated Invocation: `Cascade`

Finally, sometimes you want to compose a function with itself several times:

```
? print first bf bf bf bf [The Continuing Story of Bungalow Bill]
Bungalow
? print first (cascade 4 "bf [The Continuing Story of Bungalow Bill])
Bungalow
```

`Cascade` takes three inputs. The first is a number, indicating how many times to invoke the function represented by the second argument. The third argument is the starting value.

```
to power :base :exponent
output cascade :exponent [? * :base] 1
end

? print power 2 8
256

to range :from :to
output cascade :to-:from [sentence ? (1+last ?)] (sentence :from)
end

? show range 3 8
[3 4 5 6 7 8]
```

Like `map`, `cascade` can be used with extra inputs to deal with more than one thing at a time. One example in which multi-input `cascade` is useful is the Fibonacci sequence. Each number in the sequence is the sum of the two previous numbers; the first two numbers are 1. So the sequence starts

$$1, 1, 2, 3, 5, 8, 13, \ldots$$

A formal definition of the sequence looks like this:

$$\begin{aligned} F_0 &= 1, \\ F_1 &= 1, \\ F_n &= F_{n-1} + F_{n-2}, \qquad n \geq 2. \end{aligned}$$

In order to compute, say, F_{23}, we must know both F_{22} and F_{21}. As we work our way up, we must always remember the two most recent values, like this:

	Most recent value	Next most recent value
start	1	0
step 1	1	1
step 2	2	1
step 3	3	2
step 4	5	3
...	...	...
step 22	F_{22}	F_{21}
step 23	$F_{22}+F_{21}$	F_{22}

To express this using `cascade`, we can use `?1` to mean the most recent value and `?2` to mean the next most recent. Then at each step, we need a function to compute the new `?1` by adding the two known values, and a function to copy the old `?1` as the new `?2`:

```
to fib :n
output (cascade :n [?1+?2] 1 [?1] 0)
end
```

```
? print fib 5
8
? print fib 23
46368
```

Another situation in which multi-input `cascade` can be useful is to process every member of a list, using `?1` to remember the already-processed ones and `?2` to remember the still-waiting ones. The simplest example is reversing the words in a sentence:

```
to reverse :sent
output (cascade (count :sent)
                [sentence (first ?2) ?1] []
                [butfirst ?2] :sent)
end
```

```
? print reverse [how now brown cow]
cow brown now how
```

	`?1`	`?2`
start	`[]`	`[how now brown cow]`
step 1	`[how]`	`[now brown cow]`
step 2	`[now how]`	`[brown cow]`
step 3	`[brown now how]`	`[cow]`
step 4	`[cow brown now how]`	`[]`

Here is the general notation for multi-input `cascade`:

```
(cascade howmany function1 start1 function2 start2 ...)
```

There must be as many *`function`* inputs as *`start`* inputs. Suppose there are *n* pairs of inputs; then each of the *`function`*s must accept *n* inputs. The *`start`*s provide the initial values for `?1`, `?2`, and so on; each function provides the next value for one of those. `Cascade` returns the final value of `?1`.

A Mini-project: Mastermind

It's time to put these programming tools to work in a more substantial project. You're ready to write a computer program that plays a family of games like Mastermind^{TM}. The computer picks a secret list of colors; the human player makes guesses. (The number of possible colors can be changed to tune the difficulty of the game.) At each turn, the program should tell the player how many colors in the guess are in the correct positions in the secret list and also how many are in the list, but not at the same positions. For example, suppose the program's secret colors are

```
red green blue violet
```

and the player guesses

```
red orange yellow green
```

There is one correct-position match (red, because it's the first color in both lists) and one incorrect-position match (green, because it's second in the computer's list but fourth in the player's list).

In the program, to reduce the amount of typing needed to play the game, represent each color as a single letter and each list of colors as a word. In the example above, the computer's secret list is represented as `rgbv` and the player's guess as `royg`.

There are two possible variations in the rules, depending on whether or not color lists with duplications (such as `rgrb`, in which red appears twice) are allowed. The program will accept a true-or-false input to determine whether or not duplicates are allowed.

Here's an example of what an interaction with the program should look like:

```
? **master "roygbiv 4 "false**

What's your guess?
**royg**
You have 1 correct-position matches
and 2 incorrect-position matches.

What's your guess?
**rogy**
You have 1 correct-position matches
and 2 incorrect-position matches.

What's your guess?
**orygbv**
You must guess exactly 4 colors.

What's your guess?
**oryx**
The available colors are: roygbiv

What's your guess?
**oryr**
No fair guessing the same color twice!

What's your guess?
**oryg**
You have 0 correct-position matches
and 3 incorrect-position matches.

What's your guess?
**rbyg**
You have 1 correct-position matches
and 2 incorrect-position matches.

What's your guess?
**boyg**
You have 0 correct-position matches
and 3 incorrect-position matches.

What's your guess?
**roby**
You have 1 correct-position matches
and 3 incorrect-position matches.
```

```
What's your guess?
rybo
You have 2 correct-position matches
and 2 incorrect-position matches.

What's your guess?
ryob
You win in 8 guesses!
?
```

If you prefer, just jump in and start writing the program. But I have a particular design in mind, and you may find it easier to follow my plan. The core of my program is written sequentially, in the form of a `for` instruction that carries out a sequence of steps once for each guess the user makes. But most of the "smarts" of the program are in a collection of subprocedures that use functional programming style. That is, these procedures are operations, not commands; they merely compute and output a value without taking any actions. Pay attention to how these two styles fit together. In writing the operations, don't use `make` or `print`; each operation will consist of a single `output` instruction.

☞ The first task is for the computer to make a random selection from the available colors. Write two versions: `choose.dup` that allows the same color to be chosen more than once, and `choose.nodup` that does not allow duplication. Each of these operations should take two inputs: a number, indicating how many colors to choose, and a word of all the available colors. For example, to choose four colors from the rainbow without duplication, you'd say

```
? print choose.nodup 4 "roygbiv
briy
```

You'll find the Logo primitive `pick` helpful. It takes a word or list as its input, and returns a randomly chosen member:

```
? print pick [Pete John Roger Keith]
John
? print pick [Pete John Roger Keith]
Keith
? print pick "roygbiv
b
```

Writing `choose.dup` is a straightforward combination of `pick` and `cascade`.

`Choose.nodup` is a little harder. Since we want to eliminate any color we choose from further consideration, it's plausible to use a multi-input `cascade` sort of like this:

```
(cascade :number-wanted
         [add one color] "
         [remove that color] :colors)
```

If we always wanted to choose the first available color, this would be just like the `reverse` example earlier. But we want to choose a color randomly each time. One solution is to *rotate* the available colors by some random amount, then choose what is now the first color. To use that idea you'll need a `rotate` operation that rotates a word some random number of times, like this:

```
? rotate "roygbiv
ygbivro
? rotate "roygbiv
vroygbi
? rotate "roygbiv
bivroyg
```

You can write `rotate` using `cascade` along with the Logo primitive operation `random`. `Random` takes a positive integer as its input, and outputs a nonnegative integer less than its input. For example, `random 3` will output `0`, `1`, or `2`.

☞ The second task is to evaluate the player's guess. You'll need an operation called `exact` that takes two words as inputs (you may assume they are the same length) and outputs the number of correct-position matches, and another operation called `inexact` that computes the number of wrong-position matches. (You may find it easier to write a helper procedure `anymatch` that takes two words as inputs, but outputs the total number of matches, regardless of position.) Be sure to write these so that they work even with the duplicates-allowed rule in effect. For example, if the secret word is `rgrb` and the user guesses `yrrr`, then you must report one exact and one inexact match, not one exact and two inexact.

```
? print exact "rgrb "yrrr
1
? print inexact "rgrb "yrrr
1
? print inexact "royg "rgbo
2
```

`Exact` is a straightforward application of multi-input `map`, since you want to look at each letter of the secret word along with the same-position letter of the user's guess. My solution to `anymatch` was to use `map` to consider each of the available colors. For each color, the number of matches is the smaller of the number of times it appears in the secret word and the number of times it appears in the guess. (You'll need a helper procedure `howmany` that takes two inputs, a letter and a word, and outputs the number of times that letter occurs in that word.)

☞ Up to this point, we've assumed that the player is making legitimate guesses. A valid guess has the right number of colors, chosen from the set of available colors, and (perhaps, depending on the chosen rules) with no color duplicated. Write a predicate `valid.guessp` that takes a guess as its input and returns `true` if the guess is valid, `false` otherwise. In this procedure, for the first time in this project, it's a good idea to violate functional programming style by printing an appropriate error message when the output will be `false`.

☞ We now have all the tools needed to write the top-level game procedure `master`. This procedure will take three inputs: a word of the available colors, the number of colors in the secret word, and a `true` or `false` to indicate whether or not duplicate colors are allowed. After using either `choose.dup` or `choose.nodup` to pick the secret word, I used a `for` loop to carry out the necessary instructions for each guess.